

S2 Autumn Term Review Sheet

1. $f(x) = \begin{cases} \frac{k}{x^3} & \text{if } 3 \leq x \leq 6 \\ 0 & \text{otherwise.} \end{cases}$

(a) Find k .

$$k = 24$$

(b) Find $\mathbb{E}(X)$.

$$E(X) = 4$$

(c) Find $\text{Var}(X)$.

$$\text{Var}(X) = 24 \ln 2 - 16 \approx 0.636$$

(d) Find the median.

$$M = \frac{6\sqrt{10}}{5}$$

(e) Find $\mathbb{P}(X > 5)$.

$$\frac{11}{75}$$

(f) Find probability that each of two independent observations of X are greater than 5.

$$\frac{121}{5625}$$

2. $X \sim \text{Po}(5)$

(a) Find $\mathbb{P}(X = 4)$.

$$0.175$$

(b) Find $\mathbb{P}(3 \leq X < 7)$.

$$0.6375$$

(c) Find $\mathbb{P}(X \geq 8)$.

$$0.1334$$

3. Accidents occur with a Poisson distribution with an average of 1.5 accidents per week. Determine the probability of:

(a) More than one in a week.

$$0.4422$$

(b) Exactly 2 in a week.

$$0.2510$$

(c) Exactly 4 in fortnight.

$$0.1680$$

(d) Explain why answers to (b) and (c) differ.

(e) More than 8 in 4 weeks.

$$0.1528$$

4. $X \sim \text{Po}(35)$. By using a suitable approximation find:

(a) $\mathbb{P}(X = 35)$.

$$0.0678$$

(b) $\mathbb{P}(30 \leq X < 41)$.

$$0.647$$

5. $X \sim N(12, 2^2)$. Find:

(a) $\mathbb{P}(X = 12)$.

$$0$$

(b) $\mathbb{P}(X < 12)$.

$$0.5$$

(c) $\mathbb{P}(X > 17)$.

$$0.0062$$

(d) $\mathbb{P}(8 < X < 9)$.

$$0.044$$

6. $X \sim B(100, \frac{1}{3})$. Using an appropriate approximation find $\mathbb{P}(32 < X \leq 36)$.

$$0.3196$$

7. In a failing school, 10% of the students get expelled per year. A new headmaster has been there a year and the Governors claim that this figure has now changed.

(a) Set up a hypothesis test (at the 5% level) to test this claim on a sample of 20 students.

$$H_0 : p = \frac{1}{10}, H_1 : p \neq \frac{1}{10}$$

(b) Find the critical value(s) and hence the critical region(s) for the test.

$$\text{Crit val} = 6, \text{ Crit region} = \{6, 7, \dots, 20\}$$

(c) In practice one student from the sample of 20 were expelled in the year. Conclude the test.

$$\text{Insufficient evidence to reject } H_0$$