Permutations and Combinations Introduction

In this worksheet you will need to think about distinct rearrangements of letters. The ideas are very similar to those introduced in the previous two worksheets on tree diagrams. You are reminded that when considering the distinct rearrangements of the letters 'BAB', switching the two 'B's around does not create a new 'word'.

You are also reminded that

$$P(\text{event}) = \frac{\text{number of ways of getting what I want}}{\text{total number of possible outcomes}}.$$

1. How many distinct arrangements exist for the following strings of letters?

   (a) `A` [1]
   (b) `AB` [2]
   (c) `AA` [1]
   (d) `ABC` [6]
   (e) `AAB` [3]
   (f) `ABB` [3]
   (g) `AAA` [1]
   (h) `ABCD` [24]
   (i) `ABCC` [12]
   (j) `AABB` [6]
   (k) `ABCDE` [120]
   (l) `AAAAB` [5]
   (m) `AABBB` [this type is hugely important] [10]
   (n) `AABBC` [30]
   (o) `ABCDEF` [720]
   (p) `AABCCC` [60]
   (q) `AAAABC` [30]
   (r) `AAAABB` [15]
   (s) `MISSISSIPPI` [34650]
   (t) `ASSASSIN` [840]
   (u) A list of 26 distinct letters [26!]
   (v) A list of n distinct letters [$n!$]
   (w) A list of n letters, m of one letter and the rest another [$\frac{n!}{m!(n-m!)}$]
   (x) A list of n letters, m of one letter and all the rest distinct from each other [$\frac{n!}{m!}$]

2. (a) I need to select a committee of 6 people from a class of size 13. How many distinct committees can be chosen? [1716]
   (b) If the class comprised of 8 boys and 5 girls and the committee of 6 had to contain 3 boys and 3 girls, how many distinct committees can be chosen? [560]
   (c) If the number of boys must be less than or the number of girls, how many distinct committees can be chosen? [148]
   (d) If I just pick a committee of size 6 from the 13 without at random, what is the probability I gain 3 girls and 3 boys? [Think about previous answers] [$\frac{140}{429}$]

3. Adam and Alice are a couple. Ben and Bertha are a couple. Chris and Caroline are a couple. Xavier is single. They are queuing in a line for tickets to a show. Find the number of possible rearrangements for them in the line subject to the following conditions.

   (a) No restrictions. [5040]

   (b) Adam and Alice must be together. [1440]

   (c) All the couples must be next to each other. [192]

   (d) All the men are together. [576]

   (e) The queue is formed completely at random. What is the probability all the men are standing together? [$\frac{4}{35}$]

4. I deal 5 cards from a standard deck of cards (without replacement).

   (a) How many possible hands are there? [2598960]

   (b) How many hands contain 5 hearts? [1287]

   (c) What is the probability a random hand contains 5 hearts? [$\frac{99}{199920}$]

   (d) What is the probability of a random hand containing all the same suit? (A flush in poker.) [$\frac{99}{49980}$]

   (e) What is the probability of gaining a full house? (3 or one card and two of another, eg 3 Jacks and 2 Kings.) [$\frac{94}{4165}$]

   (f) What is the probability of gaining two pair? (Two pairs of cards and one other, eg 2 sevens, 2 eights and a Queen.) [$\frac{396}{4165}$]