Introduction To Planes

1. In this question $\mathbf{a} = \overrightarrow{OA}$, $\mathbf{b} = \overrightarrow{OB}$ and $\mathbf{c} = \overrightarrow{OC}$. Find, in the form

$$\mathbf{r} = \mathbf{a} + \lambda(\mathbf{b} - \mathbf{a}) + \mu(\mathbf{c} - \mathbf{a}),$$

the equations of the planes through the following points:

(a) $A = (0, 0, 0)$, $B = (2, 3, 1)$ and $C = (1, 2, -3)$.
(b) $A = (2, 1, 0)$, $B = (5, -1, 0)$ and $C = (1, 3, 4)$.
(c) $A = (3, -1, 2)$, $B = (-1, 2, 4)$ and $C = (0, -2, 5)$.
(d) $A = (1, -7, 2)$, $B = (-2, -2, -3)$ and $C = (0, 1, -7)$.

2. By eliminating the parameters λ and μ find the cartesian equation of each plane in the form $ax + by + cz = d$, eliminating all fractions from your answer; for example if you obtain

$$\frac{x}{3} - 2y - \frac{z}{2} = 1,$$

multiply through by 6 to obtain

$$2x - 12y - 3z = 6.$$

(a) $\mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$.

(b) i. $\mathbf{r} = \lambda \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$.

ii. Explain the significance of the value of d that you discovered.

(c) $\mathbf{r} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$.

(d) $\mathbf{r} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}$.

(e) $\mathbf{r} = \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$.

(f) $\mathbf{r} = \begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$.

(g) $\mathbf{r} = \begin{pmatrix} -3 \\ 2 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 3 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -2 \\ 4 \end{pmatrix}$.

(h) $\mathbf{r} = \begin{pmatrix} -3 \\ 4 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ 4 \\ 1 \end{pmatrix}$.

(i) i. $\mathbf{r} = \lambda \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} + \mu \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

ii. Where have you seen the x, y and z coefficients before?

Answers (I hope. . .)

1. (a) $\mathbf{r} = \lambda \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}$.

   (b) $\mathbf{r} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -2 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ 2 \\ 4 \end{pmatrix}$.

   (c) $\mathbf{r} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -4 \\ 3 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} -3 \\ -1 \\ 3 \end{pmatrix}$.

   (d) $\mathbf{r} = \begin{pmatrix} 1 \\ -7 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ 5 \\ -5 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ 8 \\ -9 \end{pmatrix}$.

2. (a) $x - y - 2z = 1$.

   (b) i. $x + y - 3z = 0$.

   ii. The plane passes through the origin.

   (c) $2x - y + 4z = 3$.

   (d) $x + 2y - z = -1$.

   (e) $x + 2y + z = 5$.

   (f) $x - z = 3$.

   (g) $5x + y - 2z = -11$.

   (h) $6x - y + 16z = -38$.

   (i) i. $(a_2b_3 - a_3b_2)x + (a_3b_1 - a_1b_3)y + (a_1b_2 - a_2b_1)z = 0$.

   ii. The vector product. . .