

Introduction To Planes

By eliminating the parameters λ and μ find the cartesian equation of each plane in the form $ax + by + cz = d$, eliminating all fractions from your answer; for example if you obtain

$$\frac{x}{3} - 2y - \frac{z}{2} = 1,$$

multiply through by 6 to obtain

$$2x - 12y - 3z = 6.$$

1. $\mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}.$

2. (a) $\mathbf{r} = \lambda \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}.$

(b) Explain the significance of the value of d that you discovered.

3. $\mathbf{r} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}.$

4. $\mathbf{r} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}.$

5. $\mathbf{r} = \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}.$

6. $\mathbf{r} = \begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}.$

7. $\mathbf{r} = \begin{pmatrix} -3 \\ 2 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 3 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -2 \\ 4 \end{pmatrix}.$

8. $\mathbf{r} = \begin{pmatrix} -3 \\ 4 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ 4 \\ 1 \end{pmatrix}.$

9. (a) $\mathbf{r} = \lambda \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} + \mu \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}.$

(b) Where have you seen the x , y and z coefficients before?

Answers (I hope...)

1. $x - y - 2z = 1$.
2. (a) $x + y - 3z = 0$.
(b) The plane passes through the origin.
3. $2x - y + 4z = 3$.
4. $x + 2y - z = -1$.
5. $x + 2y + z = 5$.
6. $x - z = 3$.
7. $5x + y - 2z = -11$.
8. $6x - y + 16z = -38$.
9. (a) $(a_2b_3 - a_3b_2)x + (a_3b_1 - a_1b_3)y + (a_1b_2 - a_2b_1)z = 0$.
(b) The vector product...