

Trigonometric Integration

Evaluate the following integrals by means of a suitable trigonometric substitution.

$$1. \int \frac{5}{4+8x^2} dx. \quad \boxed{\frac{5\sqrt{2}}{8} \tan^{-1}(\sqrt{2}x) + c}$$

$$2. \int \frac{2}{\sqrt{3-x^2}} dx. \quad \boxed{2 \sin^{-1}\left(\frac{\sqrt{3}x}{3}\right) + c}$$

$$3. \int \frac{3}{5+2x^2} dx. \quad \boxed{\frac{3\sqrt{10}}{10} \tan^{-1}\left(\frac{\sqrt{10}x}{5}\right) + c}$$

$$4. \int \frac{5}{7+3x^2} dx. \quad \boxed{\frac{5\sqrt{21}}{21} \tan^{-1}\left(\frac{\sqrt{21}x}{7}\right) + c}$$

$$5. \int \frac{a}{\sqrt{b^2+kx^2}} dx. \quad \square$$

$$6. \int \frac{1}{\sqrt{18+6x+x^2}} dx. \quad \boxed{\sinh^{-1}\left(\frac{x+3}{3}\right) + c = \ln\left(\frac{x+3}{3} + \frac{\sqrt{x^2+6x+18}}{3}\right) + c}$$

$$7. \int \frac{a}{b^2+x^2-2cx+c^2} dx. \quad \boxed{\frac{a}{b} \tan^{-1}\left(\frac{x-c}{b}\right) + k}$$

$$8. \int \sqrt{4-25x^2} dx. \quad \boxed{\frac{\sqrt{4-25x^2}}{2} x + \frac{2}{5} \sin^{-1}\left(\frac{5x}{2}\right) + c}$$

$$9. \int \sqrt{9+16x^2} dx. \quad \boxed{\frac{\sqrt{9+16x^2}}{2} + \frac{9}{8} \sinh^{-1}\left(\frac{4x}{3}\right) + c}$$

Using the substitution $t = \tan\left(\frac{x}{2}\right)$ evaluate the following integrals.

$$1. \int \frac{1}{1+\sin x+\cos x} dx. \quad \square$$

$$2. \int_{\frac{\pi}{3}}^{\frac{\pi}{6}} \frac{1}{1-\sin x} dx. \quad \square$$