Roots Of Polynomial Equations

By considering the general quadratic equation

$$ax^2 + bx + c = 0$$

we re-write it as

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0.$$

Quadratics can be factorised into two linear factors $(x-\alpha)(x-\beta)$. By equating the two we find

$$\begin{aligned}(x-\alpha)(x-\beta) &= x^2 + \frac{b}{a}x + \frac{c}{a}\\ x^2 - (\alpha+\beta)x + \alpha\beta &= x^2 + \frac{b}{a}x + \frac{c}{a}.\end{aligned}$$

So we see that the sum of the roots of a quadratic is $-\frac{b}{a}$ and the product of the roots is $\frac{c}{a}$.

By two tedious derivations similar to the one above we find that for the cubic

$$ax^3 + bx^2 + cx + d = 0$$

and the quartic

$$ax^4 + bx^3 + cx^2 + dx + e = 0$$

the following:

Quadratics	Cubics	Quartics
$\alpha+\beta = -\frac{b}{a}$	$\alpha+\beta+\gamma = -\frac{b}{a}$	$\alpha+\beta+\gamma+\delta = -\frac{b}{a}$
$\alpha\beta = \frac{c}{a},$	$\alpha\beta+\alpha\gamma+\beta\gamma = \frac{c}{a}$	$\alpha\beta+\alpha\gamma+\alpha\delta+\beta\gamma+\beta\delta+\gamma\delta = \frac{c}{a}$
	$\alpha\beta\gamma = -\frac{d}{a},$	$\alpha\beta\gamma+\alpha\beta\delta+\alpha\gamma\delta+\beta\gamma\delta = -\frac{d}{a}$
		$\alpha\beta\gamma\delta = \frac{e}{a}.$