FP1 Worksheet

Standard results:

$$\sum 1 = n, \qquad \sum r = \frac{n(n+1)}{2}, \qquad \sum r^2 = \frac{n(n+1)(2n+1)}{6}, \qquad \sum r^3 = \frac{n^2(n+1)^2}{4}.$$

Complex Numbers

1. Find $\dfrac{1}{5+2i}$ in the form $a+ib$, where a and b are constants to be found.

2. Express $(2-3i)^4$ in the form $a+ib$.

3. Find the square roots of $8-6i$.

4. We define $f(x) = z^3 - 4z^2 + 14z - 20$.
   (a) By considering f(various values) find a factor of $f(x)$. Hence factorise $f(x)$ into a linear term and a quadratic term.
   (b) Hence solve $z^3 - 4z^2 + 14z - 20 = 0$, simplifying all three of your answers.

Summing Sequences

5. Find a closed form for the sum

$$(1 \times 3) + (2 \times 4) + (3 \times 5) + \cdots + (n \times (n+2)),$$

   factorising your answer fully.

Roots of Polynomials

6. The roots of the equation $2z^2 + 3z + 5 = 0$ are α and β.
   (a) Find the values of $\alpha + \beta$ and $\alpha\beta$.
   (b) Find an equation with roots 2α and 2β.
   (c) Find an equation with roots $\alpha + 1$ and $\beta + 1$.

7. The roots of the equation $z^2 - 4z - 2 = 0$ are α and β. Find a quadratic equation with roots α^2 and β^2.

8. The roots of the cubic equation $2z^3 + 5x^2 - 3z - 2 = 0$ are α, β and γ. Find a cubic equation with roots $2\alpha + 1$, $2\beta + 1$ and $2\gamma + 1$.

Induction

9. Prove by induction (for $n \geqslant 1$) the result

$$(1\times 2\times 3)+(2\times 3\times 4)+(3\times 4\times 5)+\cdots+(n\times(n+1)\times(n+2))=\frac{1}{4}n(n+1)(n+2)(n+3).$$

10. Prove by induction (for $n \geqslant 2$) the result

$$\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right)\cdots\left(1-\frac{1}{n^2}\right)=\frac{n+1}{2n}.$$

11. A sequence is defined by $a_1 = 7$ and $a_{k+1} = 7a_k - 3$. Prove by induction that

$$a_n=\frac{13\times 7^{n-1}+1}{2}.$$