F Summer Pre Trials Harder Practice 1

1. Factorise fully the following:

   (a) $(\text{dog})^2 - 2(\text{dog})(\text{cat}) - 3(\text{cat})^2$. $(\text{dog} + \text{cat})(\text{dog} - 3\text{cat})$

   (b) $(2x + 3y)^2 - (2x + 3y)(x + y) - 6(x + y)^2$. $-x(4x + 5y)$

   (c) $(x + 1)^3 - 4(x + 1)^2$. $(x - 3)(x + 1)^2$

   (d) $x^2(2x - 1) + 3x(2x - 1)^2$. $x(2x - 1)(7x - 3)$

   (e) $4(2x - 3)^3(x + 1)^2 + 3(2x - 3)^4(x + 1)$. $5(2x - 1)(x + 1)(2x - 3)^3$

   (f) $(x^2 + 2x - 3)^2 - 4(x^2 - 2x + 1)$. $(x + 1)(x + 5)(x - 1)^2$

   (g) $(2a^2 - ab - 3b^2)^2 - 9(4a^2 - 12ab + 9b^2)$. $(a + b + 3)(a + b - 3)(2a - 3b)^2$

   (h) $(m^2 - 4mn + 4n^2) - (m^2 - mn - 2n^2)^2$. $(1 - m - n)(1 + m + n)(m - 2n)^2$

2. Solve the following

   (a) $2x^4 + 16x^2 = 18$. $x = \pm 1$

   (b) $3x^4 + 192 = 60x^2$. $x = \pm 2$ or $x = \pm 4$

   (c) $\pi x^6 + 25\pi x^3 = 54\pi$. $x = -3$ or $x = \sqrt[3]{2}$

   (d) $x + 2 = 3\sqrt{x}$. $x = 4$ or $x = 1$

3. Solve the following

   (a) $2x^9 + 5x^8 = 3x^7$. $x = 0$ or $x = \frac{1}{2}$ or $x = -3$

   (b) $4\pi x^4 = \pi x^2$. $x = \pm\frac{1}{2}$ or $x = 0$

4. Solve the following

   (a) $3^{x+1} - 3^x = 54$. $x = 3$

   (b) $2^{x+1} + 2^x = 96$. $x = 5$

   (c) $2^{x+1} = 24 + 2^{x-1}$. $x = 4$

   (d) $\sqrt{x + 3} - \sqrt{x} = 1$. $x = 1$

   (e) $\sqrt{x - 2} + \sqrt{x + 6} = \sqrt{2x + 10}$. $x = 3$ (only)

   (f) $\sqrt{2x} + \sqrt{x + 7} = \sqrt{5x + 15}$. $x = 2$ (only)

   (g) $\sqrt{x} + \sqrt{x + 3} = \sqrt{x + 8}$. $x = 1$ (only)

5. Write in standard form

   (a) $3 \times 10^a + 4 \times 10^{a-1}$. 3.4×10^a

   (b) $2 \times 10^{a+1} - 7 \times 10^a$. $1.3 \times 10^{a+1}$

   (c) $9 \times 10^{a+1} + 9 \times 10^{a-1}$. $9.09 \times 10^{a+1}$

   (d) $(7 \times 10^a) \times (8 \times 10^a)$. $5.6 \times 10^{2a+1}$

   (e) $(2 \times 10^a) \div (4 \times 10^b)$. $5 \times 10^{a-b-1}$

6. Simplify fully

(a) $\dfrac{3\times 7^n + 4\times 7^{n-1}}{5\times 7^{n+1}}$. $\boxed{\frac{5}{49}}$

(b) $\dfrac{5^n + 5^{n-2}}{13\times 5^{n-1}}$. $\boxed{\frac{2}{5}}$

(c) $\dfrac{4\times 3^{n+10} - 3^{n+9}}{55\times 3^{n+8}}$. $\boxed{\frac{3}{5}}$

(d) $\dfrac{10^{n+1} + 3\times 10^n}{26\times 10^{n-1}}$. $\boxed{5}$

7. Solve the following simultaneous equations

(a) $\begin{aligned} x + y &= a + ab - b \\ x - y &= b + ab - a \end{aligned}$. (Solve for x and y) $\boxed{(x, y) = (ab, a - b)}$

(b) $\begin{aligned} ax + y &= 1 \\ x + y &= -1 \end{aligned}$. $\boxed{(x, y) = \left(\frac{2}{a-1}, \frac{a+1}{1-a}\right)}$

(c) $\begin{aligned} T - 3mg &= 3ma \\ 5mg - T &= 5ma \end{aligned}$. (Solve for T and a.) $\boxed{(T, a) = \left(\frac{15mg}{4}, \frac{g}{4}\right)}$