F Michaelmas Simultaneous Equations

Patrons are reminded to try to use the substitution method where possible. Look for an x or y to isolate from one equation. Then substitute this into the *other* equation. For example solve

$$\begin{aligned} 3x + y &= 7 \\ 2x + 5y &= 2 \end{aligned}.$$

From the first we can see that $y = 7 - 3x$. Substituting this into the second we find $2x + 5(7 - 3x) = 2$ which solves to $x = \frac{33}{13}$. We then place this value into $y = 7 - 3x$ to discover $y = -\frac{8}{13}$. So $(x, y) = (\frac{33}{13}, -\frac{8}{13})$

Present answers in the form $(x, y) = (-1, \frac{1}{2})$. (Don't forget the brackets!)

1. $\begin{aligned} x + y &= 3 \\ x - y &= 2 \end{aligned}.$ $\boxed{(x, y) = (\frac{5}{2}, \frac{1}{2})}$

2. $\begin{aligned} x - y &= 4 \\ x + y &= 7 \end{aligned}.$ $\boxed{(x, y) = (\frac{11}{2}, \frac{3}{2})}$

3. $\begin{aligned} 2x + y &= 3 \\ x - 2y &= 2 \end{aligned}.$ $\boxed{(x, y) = (\frac{8}{5}, -\frac{1}{5})}$

4. $\begin{aligned} x + 2y &= 4 \\ 3x - 2y &= -7 \end{aligned}.$ $\boxed{(x, y) = (-\frac{3}{4}, \frac{19}{8})}$

5. $\begin{aligned} 5x - y &= 4 \\ 4x - 5y &= 0 \end{aligned}.$ $\boxed{(x, y) = (\frac{20}{21}, \frac{16}{21})}$

6. $\begin{aligned} 2x - 3y &= 5 \\ 3x + 2y &= 2 \end{aligned}.$ $\boxed{(x, y) = (\frac{16}{13}, -\frac{11}{13})}$

7. $\begin{aligned} 4x - 2y &= -9 \\ 3x + 5y &= 3 \end{aligned}.$ $\boxed{(x, y) = (-\frac{3}{2}, \frac{3}{2})}$

8. $\begin{aligned} x - 2y &= 2 \\ 3x + 2y &= 1 \end{aligned}.$ $\boxed{(x, y) = (\frac{3}{4}, -\frac{5}{8})}$

9. $\begin{aligned} 3x - y &= 3 \\ 2x + 3y &= -1 \end{aligned}.$ $\boxed{(x, y) = (\frac{8}{11}, -\frac{9}{11})}$

10. $\begin{aligned} y - 2x &= 5 \\ 5x - 7y &= 2 \end{aligned}.$ $\boxed{(x, y) = (-\frac{37}{9}, -\frac{29}{9})}$

11. $\begin{aligned} x + 3y &= 4 \\ 5x - 2y &= 6 \end{aligned}.$ $\boxed{(x, y) = (\frac{26}{17}, \frac{14}{17})}$

12. $\begin{aligned} 4x - y &= 2 \\ 3x + 4y &= 1 \end{aligned}.$ $\boxed{(x, y) = (\frac{9}{19}, -\frac{2}{19})}$

13. $\begin{aligned} 5x + 4y &= 1 \\ x - 3y &= 0 \end{aligned}.$ $\boxed{(x, y) = (\frac{3}{19}, \frac{1}{19})}$

14. $\begin{aligned} x + y &= -7 \\ 2x + 3y &= 4 \end{aligned}.$ $\boxed{(x, y) = (-25, 18)}$

15. $\begin{aligned} 2x + y &= 2 \\ x - 6y &= 1 \end{aligned}.$ $\boxed{(x, y) = (1, 0)}$

16. $\begin{aligned} 3x + 2y &= 1 \\ y - 2x &= -3 \end{aligned}.$ $\boxed{(x, y) = (1, -1)}$

17. $\begin{aligned} a - b &= 2 \\ 3a - 2b &= -4 \end{aligned}.$ $\boxed{(a, b) = (-8, -10)}$

18. $\begin{aligned} 4x + 7y &= 10 \\ 3x - y &= -2 \end{aligned}.$ $\boxed{(x, y) = (-\frac{4}{25}, \frac{38}{25})}$

19. $\begin{aligned} 2x + 3y &= 1 \\ 3x - 4y &= 2 \end{aligned}.$ $\boxed{(x, y) = (\frac{10}{17}, -\frac{1}{17})}$

20. $\begin{aligned} 5x + y &= 7 \\ 4x - \tfrac{1}{2}y &= 2 \end{aligned}.$ $\boxed{(x, y) = (\frac{11}{13}, \frac{36}{13})}$

21. $\begin{aligned} x + 5y &= 0 \\ 3x + 4y &= -1 \end{aligned}.$ $\boxed{(x, y) = (-\frac{5}{11}, \frac{1}{11})}$

22. $\begin{aligned} \frac{x + y}{2} - \frac{x - y}{3} &= 1 \\ x - 2y &= 2 \end{aligned}.$ $\boxed{(x, y) = (\frac{22}{7}, \frac{4}{7})}$

23. $\begin{array}{c} \dfrac{x+2y}{3} - \dfrac{x-3y}{7} = x \\ x + 3y = 2 \end{array}$. $\boxed{(x, y) = (\frac{23}{37}, \frac{17}{37})}$

24. $\begin{array}{c} x + ay = 0 \\ 2x + 3y = -1 \end{array}$. $\boxed{(x, y) = (\frac{a}{3-2a}, \frac{1}{2a-3})}$

25. $\begin{array}{c} kx + y = 4 \\ 2x - 3y = 2 \end{array}$. $\boxed{(x, y) = (\frac{14}{2+3k}, \frac{8-2k}{2+3k})}$

26. $\begin{array}{c} ax + 4y = 6 \\ bx - y = 5 \end{array}$. $\boxed{(x, y) = (\frac{26}{a+4b}, \frac{6b-5a}{a+4b})}$

27. $\begin{array}{c} kx + y = 1 \\ 5x - ky = m \end{array}$. $\boxed{(x, y) = (\frac{m+k}{5+k^2}, \frac{5-km}{5+k^2})}$

28. $\begin{array}{c} x + y = 1 \\ ax + by = 1 \end{array}$. $\boxed{(x, y) = (\frac{1-b}{a-b}, \frac{a-1}{a-b})}$

29. $\begin{array}{c} x + ay = 3 \\ ax + by = 4 \end{array}$. $\boxed{(x, y) = (\frac{3b-4a}{b-a^2}, \frac{4-3a}{b-a^2})}$

30. $\begin{array}{c} 4x + y = 3 \\ ax + by = c \end{array}$. $\boxed{(x, y) = (\frac{c-3b}{a-4b}, \frac{3a-4c}{a-4b})}$

31. $\begin{array}{c} x + by = 2 \\ \dfrac{x+ay}{2} - \dfrac{x-y}{3} = 1 \end{array}$. $\boxed{(x, y) = (\frac{6a+4-6b}{3a+2-b}, \frac{4}{3a+2-b})}$

32. $\begin{array}{c} \dfrac{ax+1}{2} + \dfrac{by+2}{3} = 1 \\ \dfrac{5x+1}{3} + \dfrac{ay+1}{2} = 1 \end{array}$. $\boxed{(x, y) = (\frac{3a+2b}{20b-9a^2}, \frac{3a+10}{9a^2-20b})}$

Now solve the following simultaneous equations in three unknowns.

1.

$$\begin{aligned} x + y + z &= 1 \\ 2x + 3y + z &= 6 \\ x - y + 2z &= -5 \end{aligned}$$

$x = 1, y = 2, z = -2$

2.

$$\begin{aligned} x + y + z &= 1 \\ x - y + 2z &= 2 \\ 2x + 3y + 3z &= 3 \end{aligned}$$

$x = 0, y = 0, z = 1$

3.

$$\begin{aligned} 2a + b + 3c &= -7 \\ a - b + 2c &= -4 \\ 3a + 2b - c &= 11 \end{aligned}$$

$a = 3, b = -1, c = -4$

4.

$$\begin{aligned} 2x + 3y - z &= 2 \\ 4x - y + 2z &= 5 \\ 2x + y - 3z &= -4 \end{aligned}$$

$x = 1, y = 2, z = -2$

5.

$$\begin{aligned} x - y + z &= 1 \\ 2x + 2y + 3z &= 1 \\ x - y - 4z &= 2 \end{aligned}$$

$x = 1, y =, z =$

6.

$$\begin{aligned} a + 2b + c &= 3 \\ 2a + b + c &= 1 \\ a - b + 2c &= 0 \end{aligned}$$

$a =, b =, c =$

7.

$$\begin{aligned} p + q + r &= -1 \\ 2p + q + 2r &= -1 \\ p + 3q &= 1 \end{aligned}$$

$p =, q =, r =$

8.

$$\begin{aligned} x + y - z &= 1 \\ x - 2y + 3z &= 0 \\ x - y + 2z &= -1 \end{aligned}$$

$x =, y =, z =$

9.

$$\begin{aligned} 4x - 5y + 2z &= -2 \\ 5x + 7y + 3z &= 3 \\ 2x + 3y + z &= 1 \end{aligned}$$

$x =, y =, z =$