

F Lent Diophantine Equations

Note that the set of *integers* is

$$\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}.$$

Note also that the set of *positive integers* is

$$\{1, 2, 3, 4, 5, \dots\}.$$

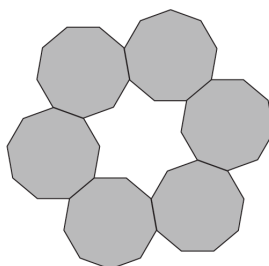
1. Find all integer solutions to $mn = 2$. $(m, n) = (2, 1), (1, 2), (-1, -2), (-2, -1)$
2. Find all integer solutions to $mn = -3$. $(m, n) = (-3, 1), (3, -1)$
3. Find all integer solutions to $(m - 5)n = 2$. $(m, n) = (7, 1), (6, 2), (4, -2), (3, -1)$
4. Find all integer solutions to $mn + m = -4$. $(m, n) = (1, -5), (-1, 3), (-2, 1), (2, -3), (-4, 0), (4, -2)$
5. Find all integer solutions to $(m - 3)(n + 10) = 1$. $(m, n) = (4, -9), (2, -11)$
6. Find all integer solutions to $mn + m + 2n = 1$. $(m, n) = (1, 0), (-1, 2), (-5, -2), (-3, -4)$
7. Find all integer solutions to $mn + 4m = 3n + 16$. $(m, n) = (7, -3), (4, 0), (5, -2), (-1, -5), (1, -6), (2, -8)$
8. Find all integer solutions to $\frac{6}{mn} + \frac{7}{n} + \frac{1}{m} + 1 = 0$. $(m, n) = (-2, -8)$ (only)
9. Find all integer solutions to $1 = \frac{1}{m} + \frac{2}{n}$. $(m, n) = (3, 3), (2, 4), (-1, 1)$ (only)
10. Find all integer solutions to $1 = \frac{3}{m} + \frac{2}{n}$. $(m, n) = (4, 8), (2, -4), (9, 3), (-3, 1), (5, 5), (1, -1), (6, 4)$
11. Find all integer solutions to $0 = 1 + \frac{3}{m} + \frac{4}{n} + \frac{11}{mn}$. □

12. Maclaurin 2015 problem.

A symmetrical ring of m identical regular n -sided polygons is formed according to the rules:

- (a) each polygon in the ring meets exactly two others;
- (b) two adjacent polygons have only an edge in common; and
- (c) the perimeter of the inner region enclosed by the ring consists of exactly two edges of each polygon.

The example in the figure shows a ring with $m = 6$ and $n = 9$. For how many different values of n is such a ring possible?



13. Maclaurin 2015 problem.

How many solutions are there to the equation $x^2 + y^2 = x^3$, where x and y are positive integers and x is less than 2011?

14. STEP problem.

(a) Find all sets of positive integers a , b and c that satisfy the equation

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 1.$$

(b) Determine the sets of positive integers a , b and c that satisfy the inequality

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \geq 1.$$