

E Lent Algebraic Fractions

Patrons are reminded that if something looks like it factorises then factorise it. And don't make the donkey-like mistake

$$\frac{2+x}{3+x} = \frac{2}{3} \quad \text{or} \quad \frac{2+x}{3+x} = \frac{3}{4}.$$

You can only cancel when there is a multiplicative relationship; for example

$$\frac{2x+2}{x^2+x} = \frac{2(x+1)}{x(x+1)} = \frac{2}{x}.$$

1. Combine the following algebraic fractions, fully simplifying your answer.

(a) $\frac{2}{x+4} + \frac{3}{x+1}$	$\frac{5x+14}{(x+1)(x+4)}$	(k) $\frac{1}{(x-1)^2} - \frac{2}{x-1}$	$\frac{3-2x}{(x-1)^2}$
(b) $\frac{3}{x-2} + \frac{5}{x+4}$	$\frac{8x+2}{(x+4)(x-2)}$	(l) $\frac{5}{x^2+3x} + \frac{2}{x+3}$	$\frac{2x+5}{x(x+3)}$
(c) $\frac{3}{x-1} - \frac{5}{x+2}$	$\frac{11-2x}{(x-1)(x+2)}$	(m) $\frac{x}{x^2-x-6} - \frac{3}{2x+4}$	$\frac{9-x}{2(x-3)(x+2)}$
(d) $\frac{1}{x-5} - \frac{3}{x-7}$	$\frac{8-2x}{(x-5)(x-7)}$	(n) $\frac{x+1}{x^2-4x+3} - \frac{x-3}{x^2-1}$	$\frac{8}{(x+1)(x-3)}$
(e) $\frac{3}{2x+4} - \frac{1}{3x+6}$	$\frac{7}{6(x+2)}$	(o) $\frac{2}{x} - \frac{3}{x^2-x} - \frac{4}{x-1}$	$-\frac{2x+5}{x(x-1)}$
(f) $\frac{a}{x+b} + \frac{a}{x+c}$	$\frac{2ax+ab+ac}{(x+b)(x+c)}$	(p) $\frac{1}{x-1} - \frac{1}{x^2+x-2} + \frac{4}{x+2}$	$\frac{5x-3}{(x-1)(x+2)}$
(g) $2 + \frac{3}{x-5}$	$\frac{2x-7}{x-5}$	(q) $\frac{3}{2x-3} + \frac{1}{x-4} - \frac{5x}{2x^2-11x+12}$	$-\frac{15}{(2x-3)(x-4)}$
(h) $\frac{5}{2-x} + 3 + \frac{1}{x+1}$	$\frac{3x^2-7x-13}{(x+1)(x-2)}$	(r) $\frac{a}{x+1} - \frac{b}{x^2+2x+1} - \frac{c}{(x+1)^3}$	$\frac{ax^2+2ax-bx+a-b-c}{(x+1)^3}$
(i) $\frac{a}{x+k} + \frac{b}{x+2k}$	$\frac{ax+bx+2ak+bk}{(x+k)(x+2k)}$		
(j) $\frac{5}{2x+3} - 2x$	$\frac{5-6x-4x^2}{2x+3}$		

2. Write the following as a single fraction.

(a) $\frac{1}{\frac{2}{x-4} - 3}$	$\frac{x-4}{14-3x}$	(g) $\frac{\frac{3}{x-2} + 2}{1 - \frac{4}{x}}$	$\frac{x(2x+1)}{(x-2)(x-4)}$
(b) $\frac{3}{\frac{1}{x-2} + \frac{2}{x}}$	$\frac{3x(x-2)}{3x+4}$	(h) $\frac{\frac{4}{3x}}{\frac{1}{x} - \frac{2}{x+1}}$	$\frac{4(x+1)}{3(1-x)}$
(c) $\frac{\frac{1}{x}}{2 - \frac{3}{x}}$	$\frac{1}{2x-3}$	(i) $\frac{\frac{1}{x-1} + \frac{3}{x-2}}{\frac{1}{x-1} - \frac{2}{x+3}}$	$\frac{(4x-5)(x+3)}{(x-2)(5-x)}$
(d) $\frac{\frac{4}{x-1}}{1 - \frac{3}{x-1}}$	$\frac{4}{x-4}$	(j) $\frac{1 - \frac{2}{x-3}}{\frac{1}{x+2} - 1}$	$-\frac{(x-5)(x+2)}{(x-3)(x+1)}$
(e) $\frac{2 + \frac{5}{x+3}}{\frac{1}{x+3} - 7}$	$-\frac{2x+11}{7x+20}$	(k) $1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{x}}}$	$\frac{3x+2}{2x+1}$
(f) $\frac{a - \frac{c}{x+b}}{e + \frac{f}{x+b}}$	$\frac{ax+ab-c}{ex+eb+f}$	(l) $3 - \frac{\frac{5}{x-1}}{x - \frac{4}{x-1}}$	$\frac{3x^2-3x-17}{x^2-x-4}$

$$(m) \frac{1}{\frac{1}{x+1} + 1} - \frac{1}{1 + \frac{1}{1+\frac{1}{x}}}. \quad \boxed{0 \text{ (ho ho)}}$$

$$(n) \frac{2}{3 + \frac{4}{5+\frac{6}{x}}} - \frac{1}{2 + \frac{3}{4+\frac{2}{x}}}. \quad \boxed{\frac{34x^2+65x+30}{(19x+18)(11x+10)}}$$

3. Cancel the following to their simplest form.

$$(a) \frac{x^2 - 1}{x^2 + 5x + 4}. \quad \boxed{\frac{x-1}{x+4}}$$

$$(f) \frac{x^2 + 6x + 8}{x^2 - 4x + 3} \div \frac{2x + 8}{x - 3}. \quad \boxed{\frac{x+2}{2(x-1)}}$$

$$(b) \frac{2x^2 + 7x - 4}{2x^2 + 9x - 5}. \quad \boxed{\frac{x+4}{x+5}}$$

$$(g) \frac{\pi x^2 - \pi x - 2\pi}{x^3 + x^2 - 6x} \div \frac{2x + 2}{3x + 9}. \quad \boxed{\frac{3\pi}{2x}}$$

$$(c) \frac{4x^2 - 8x - 12}{2x^2 - 2x - 12}. \quad \boxed{\frac{2(x+1)}{(x+2)}}$$

$$(h) \frac{12x^2 + 10x + 2}{2x - 6} \div \frac{18x^2 + 15x + 3}{5x - 15}. \quad \boxed{\frac{5}{3}}$$

$$(d) \frac{4\pi x^2 - \pi}{12\pi x^2 - 10\pi x + 2\pi}. \quad \boxed{\frac{2x+1}{2(3x-1)}}$$

$$(i) \frac{2x^2 + 2x - 4}{3x^3 - 3x} \div \frac{4x + 8}{9 + 9x}. \quad \boxed{\frac{3}{2x}}$$

$$(e) \frac{2a^2x^2 + a^2x - 10a^2}{2ax^2 - 2ax - 4a}. \quad \boxed{\frac{a(2x+5)}{2(x+1)}}$$

$$(j) \frac{4x^2 - 1}{2x^2 - 7x - 4} \div \frac{2x^2 + 5x - 3}{2x^2 - 2x - 24}. \quad \boxed{2}$$

4. Find the value of the capital letters which make the following identities correct.

$$(a) \frac{A}{x} + \frac{B}{x+1} \equiv \frac{5x+2}{x(x+1)}. \quad \boxed{A=2, B=3}$$