Simple Vectors Sheet

1. Find the equation of the line through $(1,3)$ and $(5,2)$ in vector form. $\boxed{r = \binom{1}{3} + \lambda\binom{4}{-1}}$

2. Find where
$$\mathbf{r} = \begin{pmatrix}1\\2\end{pmatrix} + \lambda\begin{pmatrix}2\\4\end{pmatrix} \text{ and } \mathbf{r} = \begin{pmatrix}3\\0\end{pmatrix} + \mu\begin{pmatrix}1\\3\end{pmatrix}$$
intercept. Why is it obvious that they must intercept? Why is this not the case with 3D lines? $\boxed{(9, 18)}$

3. Find the angle between the vectors $\binom{2}{1}$ and $\binom{-7}{3}$ using the scalar product. $\boxed{49.8^\circ \text{ (to 3 sf)}}$

4. If $\binom{1}{3}$ and $\binom{a}{-2}$ are perpendicular, find a. $\boxed{a = 6}$

5. Find where the line $\mathbf{r} = \binom{3}{4} + \lambda\binom{1}{3}$ crosses the x and y-axes. □

6. Find where the line $\mathbf{r} = \begin{pmatrix}3\\-1\\0\end{pmatrix} + \lambda\begin{pmatrix}1\\-1\\2\end{pmatrix}$ crosses the xy, xz and yz planes. $\boxed{(3, -1, 0), (2, 0, -2), (0, 2, -6)}$

7. Find the magnitude of the vector $\begin{pmatrix}2\\-1\\3\end{pmatrix}$. Hence write down a unit vector in the same direction as the original. □

8. Find three points which the line $\mathbf{r} = \begin{pmatrix}3\\-4\\-1\end{pmatrix} + t\begin{pmatrix}1\\-2\\3\end{pmatrix}$ pass through. □

9. Find the distance between the points $(1,2,-1)$ and $(0,5,-2)$. $\boxed{\sqrt{11}}$

10. Find the vector equation of the line between $(1,2,4)$ and $(3,5,1)$. Find where this line crosses the xz-plane. □

11. Convert $y = -x + 4$ to vector form. □

12. Find if the lines $\mathbf{r} = \begin{pmatrix}1\\0\\2\end{pmatrix} + \lambda\begin{pmatrix}1\\1\\2\end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix}6\\-4\\3\end{pmatrix} + \mu\begin{pmatrix}-1\\2\\1\end{pmatrix}$ intercept. $\boxed{\lambda = 2, \mu = 3, (3, 2, 6)}$

13. Explain why the lines $\mathbf{r} = \begin{pmatrix}1\\3\\2\end{pmatrix} + \lambda\begin{pmatrix}1\\1\\-2\end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix}7\\0\\2\end{pmatrix} + \mu\begin{pmatrix}-2\\-2\\4\end{pmatrix}$ never intercept. □

14. Find if the line through $(0,0,1)$ and $(2,1,3)$ ever intercepts the line $\mathbf{r} = \begin{pmatrix}1\\0\\2\end{pmatrix} + \lambda\begin{pmatrix}1\\2\\-1\end{pmatrix}$. $\boxed{\text{Skew}}$

15. By considering the scalar product on the direction vectors, find the angle between the lines $\mathbf{r} = \begin{pmatrix}1\\3\\2\end{pmatrix} + \lambda\begin{pmatrix}1\\-1\\-2\end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix}3\\-4\\0\end{pmatrix} + \mu\begin{pmatrix}1\\-2\\2\end{pmatrix}$. $\boxed{82.2^\circ}$