C4 Parametrics Proving Results

Mostly taken from the OCR C4 textbook.

1. A curve is given parametrically by the equations $x = t^2, y = t + 1$.

   (a) Find $\frac{dy}{dx}$ in terms of t and find the equation of the tangent at the point with parameter t. □

   (b) Find the cartesian equation of the curve. □

2. A curve is given parametrically by the equations $x = t^2 + t, y = t^2 - t$.

   (a) Find $\frac{dy}{dx}$ in terms of t and find the equation of the normal at the point with parameter t. □

   (b) Find the cartesian equation of the curve. □

3. (a) Find the equation of the tangent at $(-8, 4)$ to the curve which is given parametrically by $x = t^3, y = t^2$. □

   (b) Show that this tangent meets the curve again at the point with parameter 1.

   (c) Find the cartesian equation of the curve. □

4. Let P be the point on the curve $x = t^2, y = \frac{1}{t}$ with coordinates $(p^2, \frac{1}{p})$.

   (a) Find the equation of the tangent at P. $\boxed{x + 2p^3y = 3p^2}$

   (b) This tangent meets the x and y-axes at A and B respectively. Find the coordinates of both points. $\boxed{A(3p^2, 0), B(0, \frac{3}{2p})}$

   (c) Prove that $PA = 2BP$.

5. A parabola is given parametrically by $x = at^2, y = 2at$. P is the point $(ap^2, 2ap)$.

   (a) The foot of the perpendicular from P onto the axis of symmetry is F. Find the coordinates of F. □

   (b) Find the equation of the normal to the parabola at P. □

   (c) G is the point where the normal from P crosses the axis of symmetry. Find the coordinates of G. □

   (d) Prove that $FG = 2a$.

6. Let H be the curve with parametric equations $x = t, y = \frac{1}{t}$, and let P be the point on H with parameter p.

   (a) Find the equation of the tangent at P. $\boxed{x + p^2y = 2p}$

   (b) The tangent at P meets the x-axis at T. Find the coordinates of T. $\boxed{(2p, 0)}$

   (c) Prove that $OP = PT$, where O is the origin. $\boxed{OP = PT = \sqrt{\frac{p^4+1}{p^2}}}$

7. (a) For the curve

   $$x = 4at^2, \quad y = \frac{t}{a}$$

   find the equation of the tangent when $t = p$. $\boxed{8a^2py = x + 4ap^2}$

   (b) This tangent crosses the x-axis at A and the y-axis at B. Find the area of the triangle OAB where O is the origin. $\boxed{p^3}$