C4 Integration By Parts

Remember: 'Integrate the thing you are going to integrate, leave the other thing alone, take away the integral of "do both"'.

1. Evaluate

   (a) $\int xe^x \, dx.$ $\boxed{(x-1)e^x + c}$

   (b) $\int 2xe^{3x} \, dx.$ $\boxed{\frac{2xe^{3x}}{3} - \frac{2e^{3x}}{9} + c}$

   (c) $\int x \sin 2x \, dx.$ $\boxed{\frac{\sin 2x - 2x\cos 2x}{4} + c}$

   (d) $\int (1-kx)e^x \, dx.$ $\boxed{e^x(1+k-kx) + c}$

   (e) $\int \ln x \, dx.$ $\boxed{x(\ln x - 1) + c}$

   (f) $\int \theta \cos(2\theta - \pi) \, d\theta.$ $\boxed{\frac{2\theta\sin(2\theta-\pi)+\cos(2\theta-\pi)}{4} + c}$

   (g) $\int \theta \cos n\theta \, d\theta.$ $\boxed{\frac{\theta \sin n\theta}{n} + \frac{\cos n\theta}{n^2} + c}$

   (h) $\int 3x^2 e^{2x} \, dx.$ $\boxed{\frac{3e^{2x}}{4}(2x^2 - 2x + 1) + c}$

   (i) $\int ax^n \ln x \, dx.$ $\boxed{\frac{ax^{n+1}(n\ln x + \ln x - 1)}{(n+1)^2} + c}$

   (j) $\int ay^2 \cos by \, dy.$ $\boxed{\frac{ay^2 \sin by}{b} + \frac{2ay\cos by}{b^2} - \frac{2a\sin by}{b^3} + c}$

   (k) $\int (\ln x)^2 \, dx.$ $\boxed{x((\ln x)^2 - \ln x + 1) + c}$

2. A litte harder here…

   (a) $I = \int e^x \sin x \, dx.$ $\boxed{\frac{e^x(\sin x - \cos x)}{2} + c}$

   (b) $I = \int e^y \cos y \, dy.$ $\boxed{\frac{e^y(\cos y + \sin y)}{2} + c}$

   (c) $I = \int e^{kx} \sin x \, dx.$ ☐

   (d) $I = \int e^{kx} \cos px \, dx.$ $\boxed{\frac{e^{kx}(p\sin px + k\cos px)}{k^2+p^2} + c}$

3. I *definitely* like these…

   (a) $\int_0^1 \frac{2x}{e^x} \, dx.$ ☐

   (b) $\int_0^{\pi} 2\theta \sin\theta \, d\theta.$ $\boxed{2\pi}$

   (c) $\int_2^3 -xe^{x+1} \, dx.$ ☐

   (d) $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \theta \cos 2\theta \, d\theta.$ ☐

   (e) $\int_1^2 2x\sqrt{x-1} \, dx.$ [Even though a substitution is better here, please use parts.] ☐

   (f) $\int_{\pi}^{2\pi} 3\theta^2 \sin\left(\frac{\theta}{2}\right) \, d\theta.$ ☐

4. It is given that $\frac{dy}{dx} = x^3 e^{2x}$. Find the equation of the curve if it passes through the point $(0, 3)$. ☐

5. Find the area bounded by the curve $y = axe^{bx}$, the line $x = 1$ and $x = 2$. ☐

6. Find the volume formed when the curve $y = x\sqrt{\sin x}$ is rotated about the x-axis between $x = 0$ and $x = \frac{\pi}{2}$. ☐