## C3 Trigonometry Harmonic Form

1. Express  $2 \sin x + 3 \cos x$  in the form  $R \sin(x + \alpha)$  where  $0^{\circ} < \alpha < 90^{\circ}$ .

 $\sqrt{13}\sin(x + 56.3)$ 

2. Express  $\cos x - \sin x$  in the form  $R \cos(x + \alpha)$  where  $0 < \alpha < \frac{\pi}{2}$ .

 $\sqrt{2}\cos(x+\frac{\pi}{4})$ 

3. Express  $3\cos 2x + 4\sin 2x$  in the form  $R\cos(2x - \alpha)$  where  $0^{\circ} < \alpha < 90^{\circ}$ .

 $5\cos(2x - 53.1)$ 

4. Express  $\sin \theta + \sqrt{3} \cos \theta$  in the form  $R \sin(\theta + \alpha)$  where  $0 < \alpha < \frac{\pi}{2}$ .

 $2\sin(\theta + \frac{\pi}{3})$ 

5. Express  $2\cos n\theta + \frac{1}{2}\sin n\theta$  in the form  $R\cos(n\theta - \alpha)$  where  $0 < \alpha < \frac{\pi}{2}$ .

 $\frac{\sqrt{17}}{2}\cos(n\theta - 0.245)$ 

6. Express  $\sqrt{3}\sin 3\theta - \cos 3\theta$  in the form  $R\sin(3\theta - \alpha)$  where  $0^{\circ} < \alpha < 90^{\circ}$ .

7. Solve  $\cos 2\theta + 3\sin 2\theta = -1$  for  $0 < \theta < 2\pi$ .

8. By expressing  $3 \sin x + 2 \cos x$  in the form  $R \sin(x + \alpha)$ , find the maximum value of

$$3\sin x + 2\cos x$$

and the smallest positive value of x (in degrees) for which this occurs. Max value of  $\sqrt{13}$  when  $x = 56.3^{\circ}$ 

(a) What is the maximum value of  $\frac{8}{5 + 2\cos(\theta - 20)}$ ?

(b) What is the smallest positive value of  $\theta$  for which this maximum occurs?

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