

## C2 Trigonometry

Patrons are kindly reminded that

$$\sin^2 \theta + \cos^2 \theta \equiv 1 \quad \text{and} \quad \tan \theta \equiv \frac{\sin \theta}{\cos \theta}.$$

They are also kindly reminded that they should know the following:

$\theta$	$\sin \theta$	$\cos \theta$	$\tan \theta$
$30^\circ$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
$45^\circ$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
$60^\circ$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$

They are also reminded to give answers to three significant figures if answers are not exact.

1. What is the exact value of the following (without a calculator)

(a)  $\sin 300$ .

$-\frac{\sqrt{3}}{2}$

(b)  $\cos(-420)$ .

$\frac{1}{2}$

(c)  $\tan 300$ .

$-\sqrt{3}$

2. Sketch

(a)  $y = 2 \sin(3x)$  for  $0^\circ < x < 180^\circ$ .

(b)  $2y = \cos\left(\frac{x}{2}\right)$  for  $-360^\circ < x < 360^\circ$ .

3. Convert:

(a)  $\frac{\pi}{10}$  radians to degrees.

18

(b)  $20^\circ$  to radians.

$\frac{\pi}{9}$

(c) 1.2 radians to degrees.

68.8

(d)  $540^\circ$  to radians.

$3\pi$

4. Solve the following:

(a)  $2 \cos x + 1 = 0$  in range  $0^\circ < x < 360^\circ$ .

(b)  $\tan \theta = \frac{1}{\sqrt{3}}$ .

(c)  $7 \sin x = 2 \cos x$ .

(d)  $2 \sin x = 1$ .

(e)  $-\sin \theta = 0.6$ .

(f)  $\tan^2 \theta = 2$ .

$x = 120, 240$

5. Solve the following equations in the required range.

(a)  $4 \sin^2 \theta = 3$  in range  $-360^\circ < \theta < 360^\circ$ .

$\theta = \pm 60, \pm 120, \pm 240, \pm 300$

(b)  $1 + \sin x = 2 \cos^2 x$  in range  $0^\circ < x < 360^\circ$ .

$x = 30, 150, 270$

(c)  $5 + 7 \sin x = 2 \cos^2 x$  in range  $-180^\circ < x < 360^\circ$ .

$x = -150, -30, 210, 330$

(d)  $\cos^2 x + 1 = \sin^2 x + \cos x + 3$  in range  $0^\circ < x < 720^\circ$ .

$x = 180, 540$

(e)  $2 \tan^2 \theta + \tan \theta = 12 + \tan^2 \theta$  in range  $-360^\circ < \theta < 0^\circ$ .

$\theta = -288, -256, -108, -76.0$

(f)  $2 \cos^2 \theta + 14 = 8 \cos \theta - \sin^2 \theta$  in range  $-2\pi < \theta < 2\pi$ .

No solutions

6. Solve the following conversions from one trig function to another.

- (a) If  $\sin \theta = \frac{3}{4}$ , find  $\cos \theta$ .  
 (b) If  $\cos \theta = \frac{5}{6}$ , find  $\sin \theta$ .  
 (c) If  $\cos \theta = -\frac{1}{3}$ , find  $\tan \theta$ .  
 (d) If  $\sin \theta = -\frac{\sqrt{3}}{2}$ , find  $\cos \theta$ .  
 (e) If  $\cos \theta = 1$ , find  $\tan \theta$ .  
 (f) If  $\tan \theta = \frac{6}{5}$ , find  $\sin \theta$ .  
 (g) If  $\sin \theta = 0.25$ , find  $\cos \theta$  given that  $\theta$  is acute.  
 (h) If  $\sin \theta = 0.25$ , find  $\cos \theta$  given that  $\theta$  is obtuse.  
 (i) If  $\sin \theta = \frac{1}{2}$ , find  $\tan \theta$  given that  $\theta$  is obtuse.

- $\pm \frac{\sqrt{7}}{4}$   
 $\pm \frac{\sqrt{11}}{6}$   
 $\pm 2\sqrt{2}$   
 $\pm \frac{1}{2}$   
 0  
 $\pm \frac{6}{\sqrt{61}}$   
 $\frac{\sqrt{15}}{4}$   
 $-\frac{\sqrt{15}}{4}$   
 $-\frac{\sqrt{3}}{3}$

7. Solve the following equations.

- (a)  $\sin 2x = \frac{1}{2}$  for  $0^\circ < x < 360^\circ$ .  
 (b)  $2 \sin\left(\frac{x}{2}\right) = \sqrt{3}$  in range  $0 < x < 1440$ .  
 (c)  $\sin(2x - 30) = \frac{1}{\sqrt{2}}$  for  $-180^\circ < x < 180^\circ$ .  
 (d)  $\tan(3x - 10) = 0$  for  $0^\circ < x < 180^\circ$ .  
 (e)  $2 \cos(2x - 70) = 1.2$  for  $0^\circ < x < 360^\circ$ .  
 (f)  $\cos(2x + 40) = \frac{1}{2}$  in range  $0 < x < 360$ .  
 (g)  $\sin(4x - 40) = 0.6$  for  $0^\circ < x < 180^\circ$ .  
 (h)  $\sin^2(2x - 30) - 1 = 0$  for  $-180^\circ < x < 180^\circ$ .

- $x = 15, 75, 195, 255$   
 $x = 120, 240, 840, 960$   
 $x = -142.5, -97.5, 37.5, 82.5$   
 $x = \frac{10}{3}, \frac{190}{3}, \frac{370}{3}$   
 $x = 8.44, 61.6, 188, 242$   
 $x = 10, 130, 190, 310$   
 $x = 19.2, 45.8, 109, 136$   
 $x = -120, -30, 60, 150$

8. In triangle  $ABC$ ,  $AB = 7$ ,  $AC = 11$  and  $\hat{BAC} = 100^\circ$ .

- (a) Find length  $BC$ .  
 (b) Find angle  $\hat{ABC}$ .

- 14.0  
 50.6

9. In triangle  $XYZ$ ,  $XY = 11$ ,  $YZ = 6$  and  $\hat{YXZ} = 20^\circ$ .

- (a) Find angle  $\hat{XYZ}$ .  
 (b) Find the area of the triangle.

- 18.8 or 121  
 10.7 or 28.2

10. Prove the following results

(a)  $\frac{1 - 2 \sin^2 \theta}{\cos \theta + \sin \theta} \equiv \cos \theta - \sin \theta$ .

11. (a) Find the maximum value of  $2 + \sin x$ .   
 (b) Find the smallest positive value of  $x$  for which this occurs.   
 12. (a) Find the minimum value of  $5 + 2 \cos(x - 40)$ .   
 (b) Find the smallest positive value of  $x$  for which this occurs.   
 13. (a) Find the maximum value of  $2 - 7 \sin(2x - 10)$ .   
 (b) Find the smallest positive value of  $x$  for which this occurs.   
 14. (a) Find the minimum value of  $\frac{5}{7 + 3 \sin(x - 30)}$ .   
 (b) Find the smallest positive value of  $x$  for which this occurs.