

## C2 Integration Indefinite

1. Differentiate with respect to  $x$  the following:

- (a)  $3x^2 + 4x - 1$ .
- (b)  $3x^2 + 4x$ .
- (c)  $3x^2 + 4x + 2$ .
- (d)  $3x^2 + 4x + \frac{3}{2}$ .

2. The given expression is for  $\frac{dy}{dx}$ ; integrate to find  $y$ . [Don't forget the 'moral of the story' from question 1.]

(a)  $x^7$ .

$$\frac{x^8}{8} + c$$

(b)  $3x^5$ .

$$\frac{3x^6}{6} + c$$

(c)  $\frac{1}{10}x^4$ .

$$\frac{x^5}{50} + c$$

(d)  $\sqrt{x}$ .

$$\frac{2x^{3/2}}{3} + c$$

(e)  $\frac{1}{2x^3}$ .

$$-\frac{1}{4x^2} + c$$

(f)  $\frac{5}{\sqrt{x}}$ .

$$10\sqrt{x} + c$$

(g)  $(2x + 3)^2$ .

$$\frac{4}{3}x^3 + 6x^2 + 9x + c$$

(h)  $x^3 + 2x^2 - 3x - 1$ .

$$\frac{1}{4}x^4 + \frac{2}{3}x^3 - \frac{3}{2}x^2 - x + c$$

(i)  $\sqrt{x} \left(x + \frac{1}{x}\right)$ .

$$\frac{2}{5}x^{5/2} + 2x^{1/2} + c$$

(j)  $(x + 2)(x^2 - 6)$ .

$$\frac{1}{4}x^4 + \frac{2}{3}x^3 - 3x^2 - 12x + c$$

(k)  $\left(3x - \frac{1}{5x}\right)^2$ .

$$3x^3 - \frac{6}{5}x - \frac{1}{25x} + c$$

(l)  $4.8x^{-1.4}$ .

$$-12x^{-0.4} + c$$

3. Sketch the family of curves that satisfy  $\frac{dy}{dx} = -x$ .

Find the equation of the member of this family which passes through (4, 6).

4. If  $\frac{dy}{dx} = 3x^2 - 4x$  and  $y = 10$  when  $x = 1$ , find  $y$  in terms of  $x$ .

5. The gradient of a curve has the value  $x - x^2$  at the point  $(x, y)$ . If the curve passes through the point (3, 5), find the equation of the curve. Find also the equation of the tangent to the curve at the point (3, 5).

6. Find a formula for  $y$  in terms of  $x$  if  $\frac{dy}{dx} = \frac{x^2 - 1}{\sqrt{x}}$  and  $y = 0$  when  $x = 1$ .

Calculate  $y$  when  $x = 4$ .

$$\frac{52}{5}$$

7. The depth of water  $h$  metres, in the hold of the 'Slice of life<sup>1</sup>'  $t$  hours after she sprang a leak obeyed the law  $\frac{dh}{dt} = 1.2t$ . How long did it take for the depth to increase to 4 metres.

8. Obtain an expression for  $V$  given that  $\frac{dV}{dx} = (2x^2 - 3) \left(3 + \frac{1}{x^2}\right)$ .

If  $V = 10$  when  $x = 1$ , find the value of  $V$  when  $x = 3$ .

<sup>1</sup>Name the current TV show this is from...

9. It is given that  $\frac{dy}{dx}$  is inversely proportional to  $x^2$  and that  $y$  and  $\frac{dy}{dx}$  are each equal to 1 when  $x = 2$ . Express  $y$  in terms of  $x$ .
10. A curve has stationary points at the points where  $x = 0$  and  $x = 2$ . Write down an expression, of the smallest degree possible, for the gradient of the curve. Given that the curve passes through the point  $(3, 2)$  and has gradient 4 there, obtain the equation of the curve.
11. A curve has gradient  $4x - 3$  at the point  $(x, y)$  and the minimum value of  $y$  is  $-\frac{1}{8}$ . Express  $y$  in terms of  $x$  and sketch the curve.
12. (a) Sketch the graph of  $f(x) = \frac{1}{x}$ .

(b) Now consider the function  $g(x) = \frac{1 - |x|}{x}$ .

Show that

$$g(x) = \frac{1}{x} - 1 \text{ if } x > 0$$

and

$$g(x) = \frac{1}{x} + 1 \text{ if } x < 0.$$

Hence show that  $g'(x) = f'(x)$  for all non-zero values of  $x$ .

- (c) Sketch the graph of  $g$ , and check that it cannot be obtained by any translation of the graph of  $f$ .