

C2 Factor & Remainder Theorem

- The equation $f(x) = x^3 - 4x^2 + x + 6 = 0$ has three integer roots.
 - List the values of a for which it is sensible to check whether $f(a) = 0$ and check each of them.
 - Solve $f(x) = 0$.
- By considering f (various sensible values), fully factorise the following:
 - $2x^3 + 5x^2 - x - 6$. $(2x+3)(x-1)(x+2)$
 - $3x^3 - 2x^2 - 7x - 2$. $(3x+1)(x+1)(x-2)$
 - $2x^4 - 9x^3 + x^2 + 12x$. $x(x-4)(2x-3)(x+1)$
 - $6x^4 - 7x^3 - 12x^2 + 3x + 2$. $(x-2)(x+1)(2x-1)(3x+1)$
- Given that $(x - 4)$ is a factor of $2x^3 - 5x^2 - 14x + a$, find a . $a = 8$
- Given that $(x + 3)$ is a factor of $3x^3 + 9x^2 + bx + 6$, find b . $b = 2$
- Given that $(2x - 1)$ is a factor of $2x^3 + cx^2 + 8x - 2$, find c . $c = -9$
- Given that $(x + 3)$ and $(x - 2)$ are factors of $2x^3 + dx^2 + ex - 6$, find d and e . $d = 3, e = -11$
- Find the remainder when $2x^3 + 4x^2 - x - 1$ is divided by $(x + 1)$. 2
- Find the remainder when $x^3 - 2x^2 + 3x + 2$ is divided by $(2x - 1)$. $\frac{25}{8}$
- Given that the remainder when $x^3 + 3x^2 + ax - 3$ is divided by $(x - 1)$ is 3, find a . $a = 2$
- Given that the remainder when $x^3 + bx^2 + x - 3$ is divided by $(x - 3)$ is -9 , find b . $b = -4$
- Given that the remainder when $cx^3 + 4x^2 + 6x - 3$ is divided by $(2x - 1)$ is 2, find c . $c = 8$
- Given that $(x - 1)$ is a factor of $f(x) = 2x^3 + \alpha x^2 + \beta x + 2$, and that the remainder when $f(x)$ is divided by $(x - 3)$ is 14, find α and β . $\alpha = -5, \beta = 1$
- When $x^3 + ax^2 + bx + 8$ is divided by $x - 3$ the remainder is 2 and when it is divided by $x + 1$ the remainder is -2 . Find a and b and hence obtain the remainder on dividing by $x - 2$. □
- When $f(x) = 2x^3 + ax^2 + bx + 6$ is divided by $x - 1$ there is no remainder and when $f(x)$ is divided by $x + 1$ the remainder is 10. Find a and b and hence solve the equation $f(x) = 0$. $a = -1, b = -7, x = 1$ or $x = \frac{3}{2}$ or $x = -2$
- The remainder when $x^3 + ax^2 + bx - 1$ is divided by $x - 2$ is 17 more than when it is divided by $x - 1$. The remainder when $x^3 + 2x^2 + 3x - 1$ is divided by $x + 1$ is 7 less than when it is divided by $x - 1$. Find a and b . □
- The polynomial $p(x) = x^3 + ax^2 + bx + c$ leaves remainders $-36, -20, 0$ on division by $x + 1, x + 2, x + 3$ respectively. Solve the equation $p(x) = 0$. $x = -3$ or $x = -7$ or $x = 2$

17. A cuboidal tank has a square base (of side length x) and maximum volume 8 m^3 .

(a) Write down an expression, in terms of x , for the height of the tank.

(b) Show that the surface area of the tank is

$$\left(x^2 + \frac{32}{x}\right) \text{m}^2.$$

(c) Given that the surface area is 24m^2 show that $x^3 - 24x + 32 = 0$.

(d) Solve $x^3 - 24x + 32 = 0$ to find the possible values for x .

18. Show that $x - y - z$ is a factor of the expression

$$x^3 + y^3 + z^3 - yz(y + z) - zx(z + x) - xy(x + y) + 2xyz.$$

Without further working write down two other factors of this expression.

19. Factorise $x^5 - 1$.

$$(x - 1)(x^4 + x^3 + x^2 + x + 1)$$

20. Factorise $x^6 + 1$.

$$(x^2 + 1)(x^4 - x^2 + 1)$$

21. Factorise $x^5 - x^4 - 1$.

$$(x^2 - x + 1)(x^3 - x - 1)$$

22. Factorise $2x^6 - 3x^5 + x^4 - 4x^3 + 4x^2 + x - 1$.

$$(2x^2 - 3x + 1)(x^4 - 2x - 1)$$

23. Factorise $x^7 - 2x^6 + 2x^3 - 1$.

$$(x - 1)(x^3 - x - 1)(x^3 - x^2 - 1)$$

24. STEP. The polynomial $p(x)$ is of degree 9 and $p(x) - 1$ is exactly divisible by $(x - 1)^5$.

(a) Find the value of $p(1)$.

(b) Show that $p'(x)$ is exactly divisible by $(x - 1)^4$.

(c) Given also that $p(x) + 1$ is exactly divisible by $(x + 1)^5$, find $p(x)$.