

C2 Circles

Patrons are reminded that the tangent to a circle lies perpendicular to the radius *at that point*. Also remember that the angle subtended in a semi-circle is 90° .

1. Write down (or calculate) the equation of the circle with the desired properties.

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| (a) The circle with centre $(0, -2)$ and radius 4. | $x^2 + (y + 2)^2 = 16$ |
| (b) The circle with centre $(-5, 3)$ and radius 2. | $(x + 5)^2 + (y - 3)^2 = 4$ |
| (c) The circle with centre $(2, -1)$ and radius $\sqrt{2}$. | $(x - 2)^2 + (y + 1)^2 = 2$ |
| (d) The circle with centre $(1, -\frac{2}{3})$ and radius $3\sqrt{3}$. | $(x - 1)^2 + (y + \frac{2}{3})^2 = 27$ |
| (e) The circle with centre $(5, 12)$ which passes through the origin. | $(x - 5)^2 + (y - 12)^2 = 169$ |
| (f) The circle with centre $(0, 4)$ which passes through the point $(3, 0)$. | $x^2 + (y - 4)^2 = 25$ |
| (g) The circle with centre $(1, -1)$ which passes through the point $(2, 1)$. | $(x - 1)^2 + (y + 1)^2 = 5$ |
| (h) The circle with AB a diameter where $A = (1, 7)$ and $B = (4, 2)$. | $(x - \frac{5}{2})^2 + (y - \frac{9}{2})^2 = \frac{17}{2}$ |

2. Find the centre and radius of the following circles.

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| (a) $x^2 + y^2 = 49$. | Centre = $(0, 0)$, Radius = 7 |
| (b) $x^2 + y^2 = 20$. | Centre = $(0, 0)$, Radius = $2\sqrt{5}$ |
| (c) $2x^2 + 2y^2 = 7$. | Centre = $(0, 0)$, Radius = $\frac{\sqrt{14}}{2}$ |
| (d) $(x - 1)^2 + y^2 = 50$. | Centre = $(1, 0)$, Radius = $5\sqrt{2}$ |
| (e) $(x + 8)^2 + (y + 3)^2 = 1$. | Centre = $(-8, -3)$, Radius = 1 |
| (f) $(x - 3)^2 + (y + 7)^2 = 18$. | Centre = $(3, -7)$, Radius = $3\sqrt{2}$ |
| (g) $(x - \frac{2}{3})^2 + (y + \pi)^2 + 1 = 29$. | Centre = $(\frac{2}{3}, -\pi)$, Radius = $2\sqrt{7}$ |
| (h) $x^2 + y^2 - 2x = 0$. | Centre = $(1, 0)$, Radius = 1 |
| (i) $x^2 + y^2 - 10x + 14y - 7 = 0$. | Centre = $(5, -7)$, Radius = 9 |
| (j) $x^2 + y^2 + 6x - 8y + 1 = 0$. | Centre = $(-3, 4)$, Radius = $2\sqrt{6}$ |
| (k) $x^2 + y^2 + x - 3y = 3$. | Centre = $(-\frac{1}{2}, \frac{3}{2})$, Radius = $\frac{\sqrt{22}}{2}$ |
| (l) $2x^2 + 2y^2 = 2y + 8$. | Centre = $(0, \frac{1}{2})$, Radius = $\frac{\sqrt{17}}{2}$ |
| (m) $x^2 + y^2 + \alpha x + \beta y = 0$. | Centre = $(-\frac{\alpha}{2}, -\frac{\beta}{2})$, Radius = $\frac{\sqrt{\alpha^2 + \beta^2}}{2}$ |
| (n) $px^2 + py^2 + qx + ry + s = 0$. | Centre = $(-\frac{q}{2p}, -\frac{r}{2p})$, Radius = $\frac{\sqrt{q^2 + r^2 - 4p^2s}}{2p}$ |

3. Find where the line (or curve) crosses the given circle.

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| (a) $x + y = 3$ and $x^2 + y^2 = 5$. | $(1, 2)$ or $(2, 1)$ |
| (b) $y = x - 1$ and $x^2 + (y - 3)^2 = 26$. | $(5, 4)$ or $(-1, -2)$ |
| (c) $y = 2x - 1$ and $x^2 + y^2 = 1$. | $(0, -1)$ or $(\frac{4}{5}, \frac{3}{5})$ |
| (d) $y = 3x - 1$ and $(x - 1)^2 + (y - 2)^2 = 10$. | $(0, -1)$ or $(2, 5)$ |
| (e) $y = x + 1$ and $x^2 + y^2 = 9$. | $(\frac{-1+\sqrt{17}}{2}, \frac{1+\sqrt{17}}{2})$ or $(\frac{-1-\sqrt{17}}{2}, \frac{1-\sqrt{17}}{2})$ |

(f) $y = x^2 - 2$ and $x^2 + y^2 = 4$. [Don't just dive in here! Think *before* writing.]

$$(0, -2) \text{ or } (\sqrt{3}, 1) \text{ or } (-\sqrt{3}, 1)$$

4. Find where the circle $x^2 + (y - 1)^2 = 10$ crosses the x -axis.

$$(3, 0) \text{ or } (-3, 0)$$

5. Find where the circle $(x + 1)^2 + (y + 2)^2 = 12$ crosses the y -axis.

$$(0, -2 + \sqrt{11}) \text{ or } (0, -2 - \sqrt{11})$$

6. Find the required tangents or normals.

(a) Find the equation of the tangent to $x^2 + y^2 = 25$ at the point $(3, 4)$ in the form $ax + by + c = 0$.

$$3x + 4y - 25 = 0$$

(b) Find the equation of the normal to $x^2 + y^2 = 17$ at the point $(1, 4)$ in the form $ax + by + c = 0$.

$$4x - y = 0$$

(c) Find the equation of the tangent to $(x - 1)^2 + (y - 3)^2 = 2$ at the point $(2, 4)$ in the form $ax + by + c = 0$.

$$x + y - 6 = 0$$

(d) Find the equation of the tangent to $(x + 2)^2 + (y - 1)^2 = 5$ at the point $(0, 0)$ in the form $ax + by + c = 0$.

$$2x - y = 0$$

(e) Find the equation of the normal to $x^2 + y^2 - 6x + 2y = 15$ at the point $(-1, 2)$ in the form $ax + by + c = 0$. [Hint: Find the centre of the circle.]

$$3x + 4y - 5 = 0$$

(f) Find the equation of the tangent to $x^2 + y^2 + 4x + 2y = 15$ at the point $(2, 1)$ in the form $ax + by + c = 0$.

$$2x + y - 5 = 0$$

(g) Find the equation of the tangent to $x^2 + y^2 + 4x - 4y = 5$ at the point $(0, 5)$ in the form $ax + by + c = 0$.

$$2x + 3y - 15 = 0$$

(h) Find the equation of the normal to $x^2 + y^2 - x + 3y = 4$ at the point $(1, 1)$ in the form $ax + by + c = 0$.

$$5x - y - 4 = 0$$

(i) Find the equation of the tangent to $x^2 + y^2 - 5y = 25$ at the point $(-1, -3)$ in the form $ax + by + c = 0$.

$$2x + 11y + 35 = 0$$

(j) Find the equation of the normal to $x^2 + y^2 - x + 2y - 26 = 0$ at the point $(2, 4)$ in the form $ax + by + c = 0$.

$$10x - 3y - 8 = 0$$

7. Find the equations of the tangents drawn from the point $(4, -3)$ to the circle $x^2 + y^2 = 5$.

8. Find the value of k such that $(x - 3)^2 + (y + k)^2 = 16$ merely touches the x -axis.

$$k = \pm 4$$

9. Find the value(s) of k such that $y = k - x$ lies tangent to the circle $x^2 + y^2 - 2x + 2y = 1$.

$$k = \pm\sqrt{6}$$

10. Find the value(s) of k such that $y = 2x + k$ lies tangent to the circle $x^2 + y^2 - 2x + y = 1$.

11. You are given that $A = (0, 0)$, $B = (2, 2)$ and $C = (-4, 4)$.

(a) Calculate the gradients of AB , AC and BC .

$$1, -1, -\frac{1}{3}$$

(b) By considering said gradients, state (with reason) which of AB , AC and BC are perpendicular.

$$AB \text{ and } AC$$

(c) Hence state (with reason) which of AB , AC or BC is the diameter of the circle passing through A , B and C .

$$BC$$

(d) Find the equation of the circle passing through A , B and C .

$$(x + 1)^2 + (y - 3)^2 = 10$$

12. Using precisely the same method as the previous question, find the equation of the circle through the following sets of points.

(a) $R = (1, 0)$, $S = (3, 0)$, $T = (2, 1)$.

$$(x - 2)^2 + y^2 = 1$$

(b) $P = (2, 0), Q = (1, -1), R = (3, -3)$.

$$(x - \frac{5}{2})^2 + (y + \frac{3}{2})^2 = \frac{5}{2}$$

(c) $K = (2, -3), L = (-2, -1), M = (-1, 1)$.

$$(x - \frac{1}{2})^2 + (y + 1)^2 = \frac{25}{4}$$

(d) $A = (8, -1), B = (3, 4), C = (2, 1)$.

$$(x - \frac{11}{2})^2 + (y - \frac{3}{2})^2 = \frac{25}{2}$$

13. Now you've got to find a new method! Find the circle that passes through the following.

(a) $A = (0, 0), B = (2, 0), C = (4, -2)$.

$$(x - 1)^2 + (y + 3)^2 = 10$$

(b) $A = (2, 2), B = (4, 3), C = (6, 9)$.

$$(x - \frac{1}{2})^2 + (y - \frac{15}{2})^2 = \frac{65}{2}$$

(c) $U = (-1, 1), V = (2, -1), W = (-2, 0)$.

$$(x + \frac{1}{10})^2 + (y + \frac{9}{10})^2 = \frac{221}{50}$$

(d) $P = (0, 0), Q = (a, 0), R = (1, 1)$.

$$(x - \frac{a}{2})^2 + (y + \frac{a-2}{2})^2 = \frac{a^2 - 2a + 2}{2}$$