

## C2 Binomial Expansion

1. Expand and simplify the following:

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|---|--|
| (a) $(x + 2)^3$ .                                     | $x^3 + 6x^2 + 12x + 8$   |
| (b) $(3x + 1)^4$ .                                    | $81x^4 + 108x^3 + 54x^2 + 12x + 1$                                       |
| (c) $(x - 3)^4$ .                                     | $x^4 - 12x^3 + 54x^2 - 108x + 81$  |
| (d) $(x - 1)^5$ .                                     | $x^5 - 5x^4 + 10x^3 - 10x^2 + 5x - 1$                                    |
| (e) $\left(\frac{2}{x} + 1\right)^4$ .                | $\frac{16}{x^4} + \frac{32}{x^3} + \frac{24}{x^2} + \frac{8}{x} + 1$     |
| (f) $(3 - x^2)^5$ .                                   | $243 - 405x^2 + 270x^4 - 90x^6 + 15x^8 - x^{10}$                         |
| (g) $(2x^2 + 3)^4$ .                                  | $16x^8 + 96x^6 + 216x^4 + 216x^2 + 81$                                   |
| (h) $(2x - 5y)^3$ .                                   | $8x^3 - 60x^2y + 150xy^2 - 125y^3$                                       |
| (i) $(p + 3q^2)^5$ .                                  | $p^5 + 15p^4q^2 + 90p^3q^4 + 270p^2q^6 + 405pq^8 + 243q^{10}$            |
| (j) $(ab - c)^4$ .                                    | $a^4b^4 - 4a^3b^3c + 6a^2b^2c^2 - 4abc^3 + c^4$                          |
| (k) $\left(x^2 + \frac{2}{x}\right)^4$ .              | $x^8 + 8x^5 + 24x^2 + \frac{32}{x} + \frac{16}{x^4}$                     |
| (l) $\left(2 + \frac{1}{y^3}\right)^3$ .              | $8 + \frac{12}{y^3} + \frac{6}{y^6} + \frac{1}{y^9}$                     |
| (m) $(\sqrt{x} - y^2)^4$ .                            | $x^2 - 4x^{\frac{3}{2}}y^2 + 6xy^4 - 4\sqrt{xy}^6 + y^8$                 |
| (n) $\left(\frac{x^2}{2} + \frac{2y^3}{3}\right)^3$ . | $\frac{x^6}{8} + \frac{x^4y^3}{2} + \frac{2x^2y^6}{3} + \frac{8y^9}{27}$ |
| (o) $(x^2 + x + 1)^3$ .                               | $x^6 + 3x^5 + 6x^4 + 7x^3 + 6x^2 + 3x + 1$                               |
| (p) $(z^2 - 4z - 3)^4$ .                              | $z^8 - 16z^7 + 84z^6 - 112z^5 - 266z^4 + 336z^3 + 756z^2 + 432z + 81$    |

2. In each of the following, find the required coefficients. *Clearly* this should be done without doing the whole multiplication.

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|--|------------------|
| (a) Coefficient of $x^2$ in $(2x + 3)^8$ .                           | 81648            |
| (b) Coefficient of $m^4$ in $(m - 3)^7$ .                            | -945             |
| (c) Coefficient of $r^4$ in $(3r - r^2)^3$ .                         | -27              |
| (d) Coefficient of $x^8$ in $\left(1 - \frac{3x^2}{2}\right)^9$ .    | $\frac{5103}{8}$ |
| (e) Coefficient of $x^{\frac{5}{2}}$ in $(2 - 5\sqrt{x})^7$ .        | -262500          |
| (f) Coefficient of $a^3b^5$ in $\left(-2a - \frac{5}{2}b\right)^8$ . | 43750            |

3. Find the value of each of the following:

- |                        |                    |
|------------------------|--------------------|
| (a) $\binom{8}{5}$ .   | 56                 |
| (b) ${}^7C_2$ .        | 21                 |
| (c) $\binom{n}{n}$ .   | 1                  |
| (d) $\binom{p}{p-1}$ . | $p$                |
| (e) $\binom{n}{n-2}$ . | $\frac{n(n-1)}{2}$ |

4. Explain why  $\binom{42}{38}$  is the same as  $\binom{42}{4}$ .

5. Express the following in the form  $a + b\sqrt{2}$ :

(a)  $(3 + \sqrt{2})^3$ .

$45 + 29\sqrt{2}$

(b)  $(1 - 3\sqrt{2})^4$ .

$433 - 228\sqrt{2}$

6. Simplify the following (but look for shortcuts!):

(a)  $(1 + \sqrt{3})^4 + (1 - \sqrt{3})^4$ .

56

(b)  $(\sqrt{6} + \sqrt{3})^4 - (\sqrt{6} - \sqrt{3})^4$ .

$216\sqrt{2}$

7. Find the term independent of  $x$  in the following expansions:

(a)  $(x + \frac{2}{x})^4$ .

24

(b)  $(2x^2 - \frac{3}{x})^6$ .

4860

(c)  $(ax^n + \frac{b}{x^m})^{5m+5n}$ .

$(\frac{5m+5n}{5m})a^{5m}b^{5n}$

8. (a) Find the first three terms in the expansion of  $(1 + 4x)^{12}$  in ascending powers of  $x$ .

$1 + 48x + 1056x^2 + \dots$

(b) By substituting a suitable value for  $x$ , find an approximation to  $1.004^{12}$  to 3 decimal places.

1.049

9. Using the first 3 terms of  $(2 + x)^5$  (in ascending powers of  $x$ ) find  $2.001^5$  to five decimal places without using a calculator.

32.08008

10. Given that  $(2 + kx)^n = 64 - 576x + cx^2 + \dots$ , find  $n$ ,  $k$  and  $c$ .

$n = 6, k = -3, c = 2160$

11. Given that  $(1 + cx)^n = 1 + 15x + 90x^2 + \dots$ , find  $c$  and  $n$ .

$c = 3, n = 5$

12. Find the first four terms (in ascending powers of  $x$ ) in the expansion of  $(2 + 3x)(1 + x)^{10}$ .

$2 + 23x + 120x^2 + 375x^3 + \dots$

13. Find the first four terms (in ascending powers of  $x$ ) in the expansion of  $(1 - 2x)^2(1 + x)^{11}$ .

$1 + 7x + 15x^2 - 11x^3 + \dots$

14. Find the  $x^3$  coefficient in the expansion of  $(x - 3)(x + 2)^5$ .

-40

15. Find the  $x^4$  coefficient in the expansion of  $(x + 1)(x - 3)^{10}$ .

-109350

16. Find the  $x^5$  coefficient in the expansion of  $(x - 2)^2(x + 1)^8$ .

0

17. Find the  $x^6$  coefficient in the expansion of  $(2x - 1)^2(x - 2)^{10}$ .

89376

18. Find an expression for the  $x^3$  coefficient in the expansion of  $(1 + ax)(x + b)^5$ .

$10ab^3 + 10b^2$

19. Find an expression for the  $x^4$  coefficient in the expansion of  $(1 + ax)(ax - b)^7$ .

$35b^3(an - 1)$

20. Find an expression for the  $x^5$  coefficient in the expansion of  $(a + x)^2(ax - 1)^{12}$ .

$990a^5 - 792a^7 + 220a^3$

21. ★ Prove that  $\binom{n}{r-1} + 2\binom{n}{r} + \binom{n}{r+1} = \binom{n+2}{r+1}$ .

22. ★ Find the coefficient of  $x^6$  in the expansion of

$(1 - 2x + 3x^2 - 4x^3 + 5x^4)^3$ .

369