Translation Of Curves

In any curve, $y = f(x)$, if every x is replaced by $x-a$, then the original curve will translate a units to the right. For example $y = x^2$ is translated 3 units to the left to make $y = (x+3)^2 = x^2+6x+9$.

In any curve if, every y is replaced by $y - b$, then the original curve will translate b units up. For example to translate $x^2 + (y - 2)^2 = 16$ 10 units up we replace y by $y - 10$ to gain $x^2 + (y - 12)^2 = 16$.

Remember that a translation can be described by means of a translation vector. $\binom{3}{-2}$ is the translation 3 to the right and 2 down. So here we would replace x by $x - 3$ and y by $y + 2$.

1. Find the equation of the given curves after the desired translation.
   (a) $y = x^2$ after translation $\binom{1}{0}$.
   (b) $y = 2x^2$ after translation $\binom{0}{1}$.
   (c) $y = x^2 + 2x$ after translation $\binom{3}{1}$. [Multiply out your answer.]
   (d) $y = \sin x$ after translation $\binom{-90}{-1}$.
   (e) $y = 2x + 3$ after translation $\binom{10}{\frac{1}{2}}$.
   (f) $y = ax^2 + bx + c$ after translation $\binom{1}{0}$. [Multiply out your answer.]
   (g) $x^2 + y^2 = 4$ after translation $\binom{1}{2}$. [Describe the shape of the curve after the translation.]
   (h) $(x - 2)^2 + (y + 3)^2 = 16$ after translation $\binom{3}{-2}$. [Describe the shape of the curve after the translation.]
   (i) $2y + 3x = 7$ after translation $\binom{4}{-2}$.
   (j) $xy + x^2 + y^2 = 1$ after translation $\binom{1}{1}$. [Multiply out your answer.]
2. Describe fully the transformation that maps the first curve onto the second.
   (a) $y = 2x$ onto $y - 3 = 2x$.
   (b) $y = \sqrt{x}$ onto $y - 2 = \sqrt{x + 3}$.
   (c) $y = \sqrt{x - 1}$ onto $y = \sqrt{x} + 3$.
   (d) $y = x^2$ onto $y = x^2 + 4$.
   (e) $y = x^2$ onto $y = x^2 + 4x$.
   (f) $y = \frac{2}{x-3}$ onto $y = \frac{2}{x}$.
   (g) $x^2 + (y - 6)^2 = 16$ onto $(x - 6)^2 + y^2 = 16$.
3. Find the translation that maps the first quadratic onto the second.
   (a) $y = x^2$ onto $y = x^2 + 4x + 4$.
   (b) $y = x^2 + 4x$ onto $y = x^2 + 6x + 1$.
   (c) $y = x^2 - 8x + 1$ onto $y = x^2 + 2x$.
   (d) $y = x^2 + 7x$ onto $y = x^2 + x + 1$.
   (e) $y = 2x^2 + 8x - 1$ onto $y = 2x^2 + 16x - 3$.
   (f) $y = 2x^2 + 2x$ onto $y = 2x^2 + 3x - 4$.
4. (a) Differentiate $y = \sqrt{x}$.
   (b) Describe the transformation that maps $y = \sqrt{x}$ onto $y = \sqrt{x - 3}$.

(c) Using the above results, explain why the gradient of $y = \sqrt{x}$ when $x = 9$ must be the same as the gradient of $y = \sqrt{x-3}$ when $x = 12$.

(d) Hence find the equation of the tangent to the curve $y = \sqrt{x-3}$ when $x = 12$.

5. By using a similar argument to the above on the curve

$$y = \frac{1}{x},$$

find the equation of the tangent to

$$y = \frac{1}{x-10}$$

when $x = 12$.