

## C1 Surds

Given a choice between  $\sqrt{8}$  and  $2\sqrt{2}$  patrons should prefer  $2\sqrt{2}$ . Also square roots should never be left on the denominator of a fraction in a final answer.

1. Write the following in the form  $a\sqrt{b}$  where  $b$  is as small as it can be:

(a) $\sqrt{8}$ .	$2\sqrt{2}$	(h) $\sqrt{ab^4c^3d^{100}}$ .	$b^2cd^{50}\sqrt{ac}$
(b) $\sqrt{27}$ .	$3\sqrt{3}$	(i) $\sqrt[3]{54}$ .	$3\sqrt[3]{2}$
(c) $\sqrt{12}$ .	$2\sqrt{3}$	(j) $\sqrt{8} + \sqrt{18}$ .	$5\sqrt{2}$
(d) $\sqrt{45}$ .	$3\sqrt{5}$	(k) $\sqrt{12} + \sqrt{75} - \sqrt{48}$ .	$3\sqrt{3}$
(e) $\sqrt{450}$ .	$15\sqrt{2}$	(l) $\sqrt{20} + \sqrt{5} - \sqrt{45}$ .	0
(f) $\sqrt{2}\sqrt{24}$ .	$4\sqrt{3}$	(m) $\sqrt{200} - \sqrt{288} + \sqrt{128}$ .	$6\sqrt{2}$
(g) $\sqrt{a^4b^2c}$ .	$a^2b\sqrt{c}$		

2. Write the following in the form  $\sqrt{a}$ :

(a) $5\sqrt{2}$ .	$\sqrt{50}$	(h) $\sqrt{12} + 3\sqrt{3}$ .	$\sqrt{75}$
(b) $4\sqrt{3}$ .	$\sqrt{48}$	(i) $\sqrt{8} + \sqrt{2}$ .	$\sqrt{18}$
(c) $2\sqrt{5}$ .	$\sqrt{20}$	(j) $\sqrt{45} - \sqrt{20}$ .	$\sqrt{5}$
(d) $7\sqrt{6}$ .	$\sqrt{294}$	(k) $\sqrt{108} + \sqrt{12} - \sqrt{3}$ .	$\sqrt{147}$
(e) $4^2\sqrt{\bullet}$ .	$\sqrt{4^4\bullet}$	(l) $\sqrt{44} - \sqrt{99} + \sqrt{176}$ .	$\sqrt{99}$
(f) $3\sqrt[3]{x+1}$ .	$\sqrt[3]{27x+27}$	(m) $\sqrt{3a^2} + \sqrt{12b^2} + \sqrt{3a^2+3b^2-6ab}$ .	$\sqrt{27b^2}$
(g) $a\sqrt[k]{k}$ .	$\sqrt[k]{a^k k}$		

3. Expand and simplify fully:

(a) $(2\sqrt{3})^3$ .	$24\sqrt{3}$	(d) $(1 - \sqrt{8})^2$ .	$9 - 4\sqrt{2}$
(b) $(\sqrt{18})^3$ .	$54\sqrt{2}$	(e) $(4 + \sqrt{32})^2$ .	$48 + 32\sqrt{2}$
(c) $(2 - \sqrt{3})^2$ .	$7 - 4\sqrt{3}$	(f) $(3 + \sqrt{20})^3$ .	$207 + 94\sqrt{5}$

4. Rationalise the denominator and simplify fully:

(a) $\frac{4}{\sqrt{2}}$ .	$2\sqrt{2}$	(g) $\frac{2}{\sqrt{2}} + \frac{\sqrt{2}}{3}$ .	$\frac{4\sqrt{2}}{3}$
(b) $\frac{1}{\sqrt{3}}$ .	$\frac{\sqrt{3}}{3}$	(h) $\frac{3}{\sqrt{5}} - 7\sqrt{5}$ .	$\frac{-32\sqrt{5}}{5}$
(c) $\frac{25}{2\sqrt{5}}$ .	$\frac{5\sqrt{5}}{2}$	(i) $\frac{1}{\sqrt{8}} + \frac{1}{\sqrt{2}}$ .	$\frac{3\sqrt{2}}{4}$
(d) $\frac{21}{6\sqrt{7}}$ .	$\frac{\sqrt{7}}{2}$	(j) $\sqrt{\frac{1}{2}} + \sqrt{\frac{1}{4}} + \sqrt{\frac{1}{8}}$ .	$\frac{2+3\sqrt{2}}{4}$
(e) $\frac{ab}{c\sqrt{b}}$ .	$\frac{a\sqrt{b}}{c}$	(k) $\frac{1}{\sqrt{2}-1}$ .	$\sqrt{2}+1$
(f) $\left(\frac{2}{\sqrt{3}}\right)^3$ .	$\frac{8\sqrt{3}}{9}$	(l) $\frac{4}{\sqrt{3}+1}$ .	$2(\sqrt{3}-1)$

(m) $\frac{4 - \sqrt{3}}{2 + \sqrt{3}}$	$\frac{11 - 6\sqrt{3}}{11 - 6\sqrt{3}}$	(q) $\frac{a}{b + c\sqrt{k}}$	$\frac{a(b-c\sqrt{k})}{b^2-c^2k}$
(n) $\frac{1 - \sqrt{5}}{\sqrt{5} + 2}$	$\frac{3\sqrt{5} - 7}{3\sqrt{5} - 7}$	(r) $\frac{9}{\sqrt{3} - 1} + \frac{7}{\sqrt{3} + 1}$	$8\sqrt{3} + 1$
(o) $\frac{\sqrt{2} + 3}{1 - 2\sqrt{2}}$	$\frac{-1 - \sqrt{2}}{-1 - \sqrt{2}}$	(s) $\frac{\sqrt{2} + 3}{\sqrt{2} - 1} + \frac{1}{\sqrt{2}}$	$\frac{10 + 9\sqrt{2}}{2}$
(p) $\frac{\sqrt{7} + 1}{\sqrt{7} - 1}$	$\frac{4 + \sqrt{7}}{3}$	(t) $\left(\frac{1}{1 - \sqrt{2}}\right)^2 + \left(\frac{2}{3 + \sqrt{2}}\right)^2$	$\frac{191 + 74\sqrt{2}}{49}$
		(u) $\frac{69}{1 + \sqrt{2} + \sqrt{3}}$	

5. Solve the following equations and simultaneous equations:

(a) $\sqrt{3}x + 1 = 0$	$x = -\frac{\sqrt{3}}{3}$	(j) $\begin{cases} 3x + 2y = 5 \\ x - \sqrt{2}y = 4 \end{cases}$	$(x, y) = (1 + \sqrt{2}, \frac{2-3\sqrt{2}}{2})$
(b) $\sqrt{2}x - 2 = x$	$x = 2(1 + \sqrt{2})$	(k) $\begin{cases} \sqrt{2}x + y = 4 \\ x + 2y = 1 \end{cases}$	$(x, y) = (2\sqrt{2} + 1, -\sqrt{2})$
(c) $\sqrt{4}x - 3 = \sqrt{3}x$	$x = 6 + 3\sqrt{3}$	(l) $\begin{cases} \sqrt{7}x + y = 5 \\ 2x + \sqrt{7}y = -4 \end{cases}$	$(x, y) = (\frac{5\sqrt{7}+4}{5}, -\frac{4\sqrt{7}+10}{5})$
(d) $\sqrt{3}x = 4 + 2x$	$x = -8 - 4\sqrt{3}$	(m) $\begin{cases} \sqrt{5}x + \sqrt{5}y = 4 \\ 3x - \sqrt{5}y = 1 \end{cases}$	$(x, y) = (\frac{3\sqrt{5}-15}{2}, \frac{75-47\sqrt{5}}{10})$
(e) $\sqrt{5}x + 1 = x - 3$	$x = -1 - \sqrt{5}$	(n) $\begin{cases} \sqrt{3}x + y = 2 \\ x - 3y = 1 \end{cases}$	$(x, y) = (\frac{21\sqrt{3}-7}{26}, \frac{7\sqrt{3}-11}{26})$
(f) $\begin{cases} \sqrt{5}x + y = 1 \\ 3x + 2y = 2 \end{cases}$	$(x, y) = (0, 1)$	(o) $\begin{cases} \sqrt{3}x - y = 2 \\ 4x + 3y = 1 \end{cases}$	$(x, y) = (\frac{21\sqrt{3}-28}{11}, \frac{41-28\sqrt{3}}{11})$
(g) $\begin{cases} \sqrt{2}x - 2y = 0 \\ x + \sqrt{2}y = 1 \end{cases}$	$(x, y) = (\frac{1}{2}, \frac{\sqrt{2}}{4})$	(p) $\begin{cases} \sqrt{a}x - y = 1 \\ x + 3y = \sqrt{a} \end{cases}$	$(x, y) = (\frac{3a+8\sqrt{a}-3}{9a-1}, \frac{3a\sqrt{a}-a-3\sqrt{a}+1}{9a-1})$
(h) $\begin{cases} x - \sqrt{5}y = 0 \\ \sqrt{5}x + y = 1 \end{cases}$	$(x, y) = (\frac{\sqrt{5}}{6}, \frac{1}{6})$		
(i) $\begin{cases} 3x + \sqrt{2}y = 7 \\ \sqrt{2}x + 5y = -1 \end{cases}$	$(x, y) = (\frac{35+\sqrt{2}}{13}, -\frac{3+7\sqrt{2}}{13})$		