C1 Quadratics In Disguise

To spot a quadratic in disguise you are looking for an equation where the power on one of the variables is twice that on the other. For example

$$(\text{whatever})^{18} + 4(\text{whatever})^{9} - 5.$$

This can then factorise to

$$((\text{whatever})^{9} + 5)((\text{whatever})^{9} - 1),$$

or you can complete the square to

$$((\text{whatever})^{9} + 2)^{2} - 9.$$

Solve the following:

1. $x^4 - 13x^2 + 36 = 0$. $x = \pm 2$ or $x = \pm 3$
2. $x^4 - 15x^2 - 16 = 0$. $x = \pm 4$
3. $x^4 + 5x^2 + 6 = 0$. No real solutions
4. $x^6 + 7x^3 = 8$. $x = -2$ or $x = 1$
5. $2x^4 = x^2 + 1$. $x = \pm 1$
6. $x + 3 = 4\sqrt{x}$. $x = 1$ or $x = 9$
7. $12x^4 = 2x^2 + 4$. $x = \pm\frac{\sqrt{6}}{3}$
8. $2(\sin x)^2 + \sin x - 1 = 0$ in range $0 < x < 360$. $x = 270$ or $x = 30$ or $x = 150$
9. $\frac{4x^4+144}{73} = x^2$. $x = \pm\frac{3}{2}$ or $x = \pm 4$
10. $2(\cos x)^2 + (\cos x) = 6$. No real solutions
11. $x = 2\sqrt{x} + 3$. $x = 9$
12. $6x^{2/3} + 5x^{1/3} - 4 = 0$. $x = \frac{1}{8}$ or $x = -\frac{64}{27}$
13. $(x^2 - 4x + 1)^2 + (x^2 - 4x + 1) - 12 = 0$. $x = 2 \pm \sqrt{6}$
14. $2\theta + 15 = 11\sqrt{\theta}$. $\theta = 9$ or $\theta = \frac{25}{4}$
15. $x^2 + \frac{72}{x^2} = 17$. $x = \pm 3$ or $x = \pm 2\sqrt{2}$
16. $\sqrt{z} - 2\sqrt[4]{z} = 3$. $z = 81$ (only)
17. $2^{2x} - 12 \times 2^x + 32 = 0$. $x = 2$ or $x = 3$
18. $t = 4\sqrt{t} + 1$. $t = 9 \pm 4\sqrt{5}$
19. $2^{2x} + 8 = 9 \times 2^x$. $x = 0$ or $x = 3$
20. $2\sqrt{x} + \frac{9}{\sqrt{x}} = 9$. $x = \frac{9}{4}$ or $x = 9$
21. $\left(\frac{1}{x}\right)^2 + 1 = 8\left(\frac{1}{x}\right)$. [Do this question in two ways.] $x = 4 \pm \sqrt{15}$
22. $2(\cos\theta)^2 = 8\cos\theta + 21$. No real solutions
23. $x^8 + 4x^4 = 3$. $x = \pm\sqrt[4]{-2 + \sqrt{7}}$
24. $2^{2x} + 1 = 2^{x+1}$. $x = 0$
25. $2^{2x} + 128 = 3 \times 2^{x+3}$. $x = 3$ or $x = 4$
26. $729 + 3^{2x+1} = 4 \times 3^{x+2}$. $x = 2$ or $x = 3$
27. $a^{2x} + a^4 = a^{x+1} + a^{x+3}$. $x = 1$ or $x = 3$

Only attempt the following if you have studied logarithms.

28. $2^{2x} - 13 \times 2^x + 42 = 0$. $x = \log_2 7$ or $x = \log_2 6$
29. $4^{2x} - 9 \times 4^x + 14 = 0$. $x = \log_4 7$ or $x = \frac{1}{2}$
30. $3^{2x} + 10 = 7 \times 3^x$. $x = \log_3 2$ or $x = \log_3 5$
31. $2^{2x} - 5 \times 2^{x+1} + 25 = 0$. $x = \log_2 5$ (repeated)
32. $3^{2x} - 3^{x+2} + 20 = 0$. $x = \log_3 4$ or $x = \log_3 5$