

C1 Quadratic Inequalities (& Beyond...)

Solve the following inequalities. Remember that if the answer lies in one region of the number line it is stated in the form “smallest $< x <$ largest”. If it is two regions it is “ $x <$ smallest **OR** $x >$ biggest”.

You must use the word ‘**OR**’! Not the word ‘and’.

Also remember that the type of the inequality sign is preserved.

Therefore questions involving $<$ or $>$ have $<$ and/or $>$ in the answer.

And questions involving \leq or \geq have \leq and/or \geq in the answer.

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|---|--|
| 1. $(x - 3)(x + 5) > 0$. Do not multiply out! | $x < -5$ or $x > 3$ |
| 2. $(x - 6)(x - 7) \leq 0$. Do not multiply out! | $6 < x \leq 7$ |
| 3. $x(x + 3) > 0$. Do not multiply out! | $x < -3$ or $x > 0$ |
| 4. $(x + 7)(x - 1) > x^2$. | $x > \frac{7}{6}$ |
| 5. $x^2 < x + 6$. | $-2 < x < 3$ |
| 6. $x^2 - 2x - 35 \geq 0$. | $x \leq -5$ or $x \geq 7$ |
| 7. $-x^2 \leq 12x + 32$. | $x \leq -8$ or $x \geq -4$ |
| 8. $x^2 > x$. | $x < 0$ or $x > 1$ |
| 9. $0 \leq -x^2 - 15x - 50$. | $-10 \leq x \leq -5$ |
| 10. $-x^2 + 7x > 12$. | $3 < x < 4$ |
| 11. $x^2 \geq 49$. | $x \leq -7$ or $x \geq 7$ |
| 12. $-x^2 \geq 5x$. | $-5 \leq x \leq 0$ |
| 13. $x^2 + 45 < -14x$. | $-9 < x < -5$ |
| 14. $5x^2 - 12 > 11x$. | $x < -\frac{4}{5}$ or $x > 3$ |
| 15. $7x^2 \leq 4x$. | $0 \leq x \leq \frac{4}{7}$ |
| 16. $6x^2 > x + 1$. | $x < -\frac{1}{3}$ or $x > \frac{1}{2}$ |
| 17. $-10x^2 + 17x + 6 \geq 0$. | $-\frac{3}{10} \leq x \leq 2$ |
| 18. $45x^2 < 450x$. | $0 < x < 10$ |
| 19. $10x^2 + 35 > 75x$. | $x < \frac{1}{2}$ or $x > 7$ |
| 20. $\pi x^2 + 3\pi x > 10\pi$. | $x < -5$ or $x > 2$ |
| 21. $20\pi x^2 > 10\pi x + 60\pi$. | $x < -2$ or $x > \frac{3}{2}$ |
| 22. $5x^2 \leq 6$. | $-\frac{\sqrt{30}}{5} \leq x \leq \frac{\sqrt{30}}{5}$ |
| 23. $x^2 - 2x - 5 > 0$. | $x > 1 + \sqrt{6}$ or $x < 1 - \sqrt{6}$ |
| 24. $x^2 > 3 - x$. | $x < \frac{-1 - \sqrt{13}}{2}$ or $x > \frac{-1 + \sqrt{13}}{2}$ |

25. $3x^2 + 15x < 6$.

$$\frac{-5-\sqrt{33}}{2} < x < \frac{-5+\sqrt{33}}{2}$$

Now try these rather harder questions (a sketch, like above, is vital).

26. $(x + 1)(x + 3)(x + 5) > 0$.

$$-5 < x < -3 \text{ or } x > -1$$

27. $(2x + 1)(x - 4)(x - 1) < 0$.

$$x < -\frac{1}{2} \text{ or } 1 < x < 4$$

28. $-(x + 3)(4 - x)(5 - x) \geq 0$.

$$x \leq -3 \text{ or } 4 \leq x \leq 5$$

29. $0 > x(x - 2)(2x + 1)$.

$$x < -\frac{1}{2} \text{ or } 0 < x < 2$$

30. $x(2x + 3)(5x - 2) \geq 0$.

$$-\frac{3}{2} \leq x \leq 0 \text{ or } x \geq \frac{2}{5}$$

31. $(x - 1)(x - 2)(x - 3)(x - 4) < 0$.

$$1 < x < 2 \text{ or } 3 < x < 4$$

32. $x(x - 2)^2(x - 9) \geq 0$.

$$x \leq 0 \text{ or } x = 2 \text{ or } x \geq 9$$

33. $(x - 2)^2(x + 3)^2 \leq 0$.

$$x = 2 \text{ or } x = -3$$

34. $\cos x < 0$ for $-360^\circ < x < 360^\circ$.

$$-270^\circ < x < -90^\circ \text{ or } 90^\circ < x < 270^\circ$$

35. $\sin x \geq \frac{1}{2}$ for $0^\circ \leq x \leq 720^\circ$.

$$30^\circ \leq x \leq 150^\circ \text{ or } 390^\circ \leq x \leq 510^\circ$$

36. $\tan x \geq \sqrt{3}$ for $0^\circ \leq x \leq 260^\circ$.

$$60^\circ \leq x < 90^\circ \text{ or } 240^\circ \leq x \leq 260^\circ$$