

C1 Graphical Transformations Worksheet

For a curve, $y = f(x)$, we have the following transformations:

- $y = af(x)$: A stretch (with x -axis invariant) parallel to the y -axis of scale factor a .
- $y = f(x) + a$: A translation of a units in the positive y direction (translation vector $\begin{pmatrix} 0 \\ a \end{pmatrix}$).
- $y = f(ax)$: A stretch (with y -axis invariant) parallel to the x -axis of scale factor $\frac{1}{a}$.
- $y = f(x + a)$: A translation of a units in the negative x direction (translation vector $\begin{pmatrix} -a \\ 0 \end{pmatrix}$).
- $y = -f(x)$: A reflection in the x -axis.
- $y = f(-x)$: A reflection in the y -axis.

Whenever you see a quadratic it is usually advantageous to complete the square.

Questions

1. Describe the transformation(s) that map the first curve onto the second:

(a) $y = 3x + 1$ onto:

- i. $y = -3x - 1$.
- ii. $y = 6x + 2$.
- iii. $y = 6x + 1$.
- iv. $y = -3x + 1$.

- Reflection in x -axis
- Stretch, sf 2 parallel to y -axis
- Stretch, sf $\frac{1}{2}$ parallel to x -axis
- Reflection in y -axis

(b) $y = x^2 + 1$ onto:

- i. $y = x^2 - 1$.
- ii. $y = x^2 + 2x + 3$.
- iii. $y = 2x^2 + 2$.
- iv. $y = 4x^2 + 1$.
- v. $y = -x^2 - 1$.

- Translation by $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$
- Translation by $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$
- Stretch, sf 2 parallel to y -axis
- Stretch, sf $\frac{1}{2}$ parallel to x -axis
- Reflection in x -axis

(c) $y = \frac{2}{x-1}$ onto:

- i. $y = \frac{2}{x}$.
- ii. $y = \frac{1}{x-1}$.
- iii. $y = \frac{2}{x-4} + 2$.
- iv. $y = \frac{2}{3x-1}$.
- v. $y = -\frac{2}{x+1}$.

- Translation by $\begin{pmatrix} -1 \\ 0 \end{pmatrix}$
- Stretch, sf $\frac{1}{2}$ parallel to y -axis
- Translation by $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$
- Stretch, sf $\frac{1}{3}$ parallel to x -axis
- Reflection in y -axis

(d) $y = \sqrt{x}$ onto:

- i. $y = \sqrt{x+3} - 2$.
- ii. $y = \sqrt{-x}$.
- iii. $y = -\sqrt{x}$.
- iv. $y = \sqrt{\frac{x}{4}}$.
- v. $y = \pi\sqrt{x}$.

- Translation by $\begin{pmatrix} -3 \\ -2 \end{pmatrix}$
- Reflection in y -axis
- Reflection in x -axis
- Stretch, sf 4 parallel to x -axis
- Stretch, sf π parallel to y -axis

2. Describe *two* possible single transformations that map:

- (a) $y = x^2$ onto $y = 9x^2$.
- (b) $y = 2^x$ onto $y = 2^{x+1}$.
- (c) $y = \sqrt{x}$ onto $y = \sqrt{3x}$.

3. Find the equation of new curves after the following transformations:

(a) $y = x^2 + 3x - 1$ after:

i. the translation $\begin{pmatrix} 0 \\ 5 \end{pmatrix}$.

$$y = x^2 + 3x + 4$$

ii. the translation $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$.

$$y = x^2 - x - 3$$

iii. the translation $\begin{pmatrix} -3 \\ -4 \end{pmatrix}$.

$$y = x^2 + 9x + 13$$

iv. a stretch parallel to the y -axis, scale factor 5.

$$y = 5x^2 + 15x - 5$$

v. a stretch parallel to the x -axis, scale factor 3.

$$y = \frac{x^2}{9} + x - 1$$

(b) $y = -x^2 + x + 1$ after:

i. a reflection in the x -axis.

$$y = x^2 - x - 1$$

ii. a reflection in the y -axis.

$$y = -x^2 - x + 1$$

iii. a stretch parallel to the y -axis, scale factor $\frac{1}{3}$.

$$y = -\frac{x^2}{3} + \frac{x}{3} + \frac{1}{3}$$

(c) $y = x + \frac{1}{x+3}$ after:

i. a stretch parallel to the y -axis scale factor 4.

$$y = 4x + \frac{4}{x+3}$$

ii. a reflection in the x -axis.

$$y = -x - \frac{1}{x+3}$$

iii. a translation $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$.

$$y = x + \frac{1}{x} - 4$$

iv. a stretch parallel to the x -axis scale factor 2.

$$y = \frac{x}{2} + \frac{2}{x+6}$$

(d) $y = x^3 + x^2 + x$ after:

i. a stretch parallel to the x -axis scale factor 2.

$$y = \frac{x^3}{8} + \frac{x^2}{4} + \frac{x}{2}$$

ii. a reflection in the x -axis.

$$y = -x^3 - x^2 - x$$

iii. the translation $\begin{pmatrix} -3 \\ 4 \end{pmatrix}$.

$$y = (x+3)^3 + (x+3)^2 + (x+3) + 4$$

iv. a reflection in the y -axis.

$$y = -x^3 + x^2 - x$$

v. the translation $\begin{pmatrix} a \\ 0 \end{pmatrix}$.

$$y = (x-a)^3 + (x-a)^2 + (x-a)$$

(e) $y = \sqrt{2x+3}$ after:

i. the translation $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$.

$$y = \sqrt{2x+5} + 2$$

ii. a stretch parallel to the y -axis scale factor 3.

$$y = 3\sqrt{2x+3}$$

iii. a reflection in the x -axis.

$$y = -\sqrt{2x+3}$$

iv. a reflection in the y -axis.

$$y = \sqrt{3-2x}$$

v. a stretch parallel to the x -axis scale factor 2.

$$y = \sqrt{x+3}$$

4. (a) Sketch the graph $y = \begin{cases} x & 0 \leq x < 1 \\ 2-x & 1 \leq x \leq 2 \\ 0 & \text{otherwise.} \end{cases}$

(b) Sketch $y = f(x+1) - 1$.

(c) Sketch $y = f\left(\frac{x}{3}\right)$.

(d) Sketch $y = -2f(x-\pi)$.

(e) The equation $f(x) = k$ has an infinite number of solutions. Find the possible value(s) of k .

5. (a) Sketch the graph $y = \begin{cases} x(x+2) & -2 \leq x < 0 \\ x(2-x) & 0 \leq x \leq 2 \\ 0 & \text{otherwise.} \end{cases}$

(b) Sketch $y = f(x-3) + 1$.

- (c) Sketch $y = 2f(x + 1)$.
- (d) Sketch $y = -f\left(\frac{x}{2}\right)$.
- (e) Sketch $y = f(-2x)$.
- (f) The equation $f(x) = k$ has exactly two distinct solutions. Find the possible value(s) of k .