Yet More Practice of Differentiation and Indices

1. Simplify:

   (a) 2^{-5}. $\boxed{\frac{1}{32}}$

   (b) $4^{-2.5}$. $\boxed{\frac{1}{32}}$

   (c) $3\sqrt{3} \times 3^{\frac{1}{2}}$. $\boxed{9}$

   (d) $\sqrt{25} \times 5^{-2}$. $\boxed{\frac{1}{5}}$

   (e) $7^{\frac{1}{2}} \times 7^{-\frac{1}{2}}$. $\boxed{1}$

   (f) $(5^4 \times 5^6)^{\frac{1}{2}}$. $\boxed{5^5}$

   (g) $\sqrt[3]{243 \times 81}$. $\boxed{27}$

   (h) $(\sqrt{27})^4$. $\boxed{729}$

   (i) $64^{\frac{1}{2}} - 64^{\frac{1}{3}}$. $\boxed{4}$

   (j) $(11^2)^3 - (11^3)^2$. $\boxed{0}$

   (k) $2^0 + 2^{-1} + 2^{-2} + 2^{-3} + 2^{-4}$. $\boxed{\frac{31}{16}}$

2. Simplify (by rationalising the denominator where necessary):

   (a) $(2 + 3\sqrt{5})^2$. $\boxed{49 + 12\sqrt{5}}$

   (b) $(2 + 2\sqrt{2})(2 - 3\sqrt{2})$. $\boxed{-8 - 2\sqrt{2}}$

   (c) $\frac{5}{\sqrt{5}}$. $\boxed{\sqrt{5}}$

   (d) $\frac{9}{3\sqrt{3}}$. $\boxed{\sqrt{3}}$

   (e) $\frac{2+\sqrt{2}}{\sqrt{2}}$. $\boxed{\sqrt{2} + 1}$

   (f) $\frac{3+\sqrt{2}}{4-\sqrt{2}}$. $\boxed{1 + \frac{1}{2}\sqrt{2}}$

   (g) $\frac{2}{\sqrt{7}-1}$. $\boxed{\frac{\sqrt{7}+1}{3}}$

3. Differentiate:

   (a) $y = x(x + 1)(x + 2)$. $\boxed{3x^2 + 6x + 2}$

   (b) $y = \sqrt{3}x^2 + \sqrt{5}x - \sqrt{2}$. $\boxed{2\sqrt{3}x + \sqrt{5}}$

   (c) $y = \frac{x^2+x}{\sqrt{x}}$. $\boxed{\frac{3x+1}{2\sqrt{x}}}$

4. Find the equation of the normal to the curve $y = x^2 + 2x + 3$ when $x = \frac{1}{2}$. $\boxed{4x + 12y = 53}$

5. Find the turning points on the curve $y = \dfrac{2x^4 - 3x^3 - 72x^2 + 8x}{x}$. $\boxed{(4, -200) \text{ and } (-3, 143)}$

6. Find the points on the curve $y = \dfrac{2x^5 - 18x^4 + 50x^3 + 14x^2}{2x^2}$ with gradient 1. $\boxed{(2, 29) \text{ and } (4, 27)}$