Differentiation Worksheet 1

1. Find the equation of the tangent to the given curve at the given point on the curve:

   (a) $y = x^2 - 2$ where $x = 1$. $\boxed{y = 2x - 3}$

   (b) $y = x^2 + 3x - 1$ where $x = 0$. $\boxed{y = 3x - 1}$

   (c) $y = \frac{1}{x}$ where $x = -1$. $\boxed{y = -x - 2}$

   (d) $y = (x - 2)(x^2 + 1)$ where $x = -1$. $\boxed{y = 8x + 2}$

   (e) $y = x^2 - 5x + 2$ where $x = 3$. $\boxed{y = x - 7}$

   (f) $y = x^2 - 2$ where $x = 0$. $\boxed{y = -2}$

2. Find the equation of the normal to the given curve at the given point on the curve:

   (a) $y = x^2 - 2$ where $x = 1$. $\boxed{x + 2y + 1 = 0}$

   (b) $y = x^2 + 3x - 1$ where $x = 0$. $\boxed{x + 3y + 3 = 0}$

   (c) $y = \frac{1}{x}$ where $x = -1$. $\boxed{y = x}$

   (d) $y = (x - 2)(x^2 + 1)$ where $x = -1$. $\boxed{x + 8y + 49 = 0}$

   (e) $y = x^2 - 5x + 2$ where $x = 3$. $\boxed{x + y + 1 = 0}$

   (f) $y = x^2 - 2$ where $x = 0$. $\boxed{x = 0}$

3. Find the equation of the normal to the curve $y = x^2 + 3x - 2$ at the point where the curve cuts the y-axis. $\boxed{x + 3y + 6 = 0}$

4. Find the equation of the tangent to the curve $y = x^2 + 5x - 2$ at the point where this curve cuts the line $x = 4$. $\boxed{y = 13x - 18}$

5. Find the equations of the tangents to the curve $y = (2x - 1)(x + 1)$ at the points where the curve cuts the x-axis. Find the point of intersection of these tangents.

   $\boxed{6x - 2y - 3 = 0 \text{ and } 3x + y + 3 = 0.\ \left(-\frac{1}{4}, -\frac{9}{4}\right)}$

6. Find the equations of the normals to the curve $y = x^2 - 5x + 6$ at the points where the curve cuts the x-axis. $\boxed{y = -x - 2 \text{ and } y = x - 3}$

7. Find the equations of the tangents to the curve $y = 3x^2 + 5x - 1$ at the points of intersection of the curve and the line $y = x - 1$. $\boxed{y = 5x - 1 \text{ and } 9x + 3y + 19 = 0}$

8. Find the coordinates of the point on $y = x^2$ at which the gradient is 2. Hence find the equation of the tangent to $y = x^2$ whose gradient is 2. $\boxed{y = 2x - 1}$

9. Find the coordinates of the point on $y = x^2 - 5$ at which the gradient is 3. Hence find the value of c for which the line $y = 3x + c$ is a tangent to $y = x^2 - 5$. $\boxed{c = -\frac{29}{4}}$

10. Find the equation of the normal to $y = x^2 - 3x + 2$ which has a gradient of $\frac{1}{2}$. $\boxed{x - 2y + 1 = 0}$

11. Find the equation of the tangent to $y = 2x^2 - 3x$ which has gradient of 1. $\boxed{y = x - 2}$

12. Find the value of k for which $y = 2x + k$ is a normal to $y = 2x^2 - 3$. $\boxed{k = -\frac{87}{32}}$

13. Find the equation of the tangent to $y = (x - 5)(2x + 1)$ which is parallel to the x-axis.

    $\boxed{y = -\frac{121}{8}}$