

## Differentiation & The Second Derivative

1. (a) Given that  $y = x^3 - 4x^2 + 5x - 2$ , find  $\frac{dy}{dx}$ .  $3x^2 - 8x + 5$   
 The point P is on the curve and its  $x$  coordinate is 3.
- (b) Calculate the  $y$  coordinate of P.  $(3, 4)$
- (c) Calculate the gradient at P.  $8$
- (d) Find the equation of the tangent at P.  $y = 8x - 20$
- (e) Find the equation of the normal at P.  $x + 8y = 35$
- (f) Find the coordinates on the curve where the gradient is 5.  $(0, -2)$  and  $(\frac{8}{3}, \frac{50}{27})$
2. (a) Find the points of intersection of  $y = 2x^2 - 9x$  and  $y = x - 8$ .  $(1, -7)$  and  $(4, -4)$
- (b) Find  $\frac{dy}{dx}$  for the curve and hence find the equation of the tangent to the curve at each of the points in (a).  $y = -5x - 2$  and  $y = 7x - 32$
- (c) Find the point of intersection of the two tangents.
3. Find any turning points on the following curves and identify their nature.
- (a)  $y = x^2 + 2x + 4$ . [Check your answer here by completing the square.]  $(-1, 3)$
- (b)  $y = 6x - x^2$ . ['Ditto']  $(3, 9)$
- (c)  $y = 2x^2 - 5x + 1$ . ['Ditto']  $(\frac{5}{4}, -\frac{9}{4})$
- (d)  $y = x^3 - 3x$ .
- (e)  $y = 4x^5 - 5x^4$ .
- (f)  $y = x^4 + x^3 - 2x^2 - 3x + 1$ .
4. You are given that  $y = x^4 - 8x^2$ .
- (a) Find  $\frac{dy}{dx}$ .
- (b) Find  $\frac{d^2y}{dx^2}$ .
- (c) Find any turning points and determine their nature.
- (d) Hence sketch the curve.
5. Given that  $y = x^5 - 3$ ;
- (a) find  $\frac{dy}{dx}$ .
- (b) find the coordinates of any stationary points and determine their nature.
- (c) sketch the curve.
6. Given that  $y = x^2(x - 2)^2$ ;
- (a) multiply out the RHS and find  $\frac{dy}{dx}$ .
- (b) find the position and nature of any turning points.
- (c) sketch the curve.
7. Given the function  $y = 3x^4 + 4x^3$ ;
- (a) find  $\frac{dy}{dx}$ .
- (b) show that the graph of the function has stationary points at  $x = 0$  and  $x = -1$  and find their coordinates.
- (c) determine whether each of the stationary points is a maximum, minimum or point of inflection, giving reasons for your answers.
- (d) Sketch the graph of the function  $y$ , giving the coordinates of the stationary points and the points where the curve crosses the axes.