

C1 Completing The Square

Patrons are reminded that if they see an expression such as $x^2 - 5 - 3x + ax + kx^2 + \pi$, the x^2 , x and 'constant' terms can be collected by factoring to yield $(1+k)x^2 + (a-3)x + (\pi-5)$. Thank you for listening.

1. Complete the square in the following quadratic expressions.

(a) $x^2 + 8x + 10.$	$(x+4)^2 - 6$	(j) $3x^2 + 4x.$	$3(x + \frac{2}{3})^2 - \frac{4}{3}$
(b) $x^2 - 20x + 110.$	$(x-10)^2 + 10$	(k) $-x^2 + 5x + \frac{1}{3}.$	$-(x - \frac{5}{2})^2 + \frac{79}{12}$
(c) $x^2 - 6x + 9.$	$(x-3)^2$	(l) $-2z^2 + \frac{1}{2}z - \frac{5}{4}.$	$-2(z - \frac{1}{8})^2 - \frac{39}{32}$
(d) $x^2 + 3x - 1.$	$(x + \frac{3}{2})^2 - \frac{13}{4}$	(m) $x^2 + ax + x + 3.$	$(x + \frac{a+1}{2})^2 + \frac{11-a^2-2a}{4}$
(e) $x^2 - x + \frac{2}{3}.$	$(x - \frac{1}{2})^2 + \frac{5}{12}$	(n) $ax^2 + bx + c.$	$a(x + \frac{b}{2a})^2 + \frac{4ac-b^2}{4a}$
(f) $3x^2 + 18x - 1.$	$3(x+3)^2 - 28$	(o) $-2ax^2 - ax + \frac{1}{a}.$	$-2a(x + \frac{1}{4})^2 + \frac{a^4+8}{8a}$
(g) $5x^2 - 20x - 3.$	$5(x-2)^2 - 23$	(p) $3x^2 + \pi x - 2x.$	$3(x + \frac{\pi-2}{6})^2 - \frac{(\pi-2)^2}{12}$
(h) $-y^2 + 8y - 20.$	$-(y-4)^2 - 4$	(q) $kz^2 + z^2 + kz - 1.$	$(k+1)(z + \frac{k}{2(k+1)})^2 - \frac{(k+2)^2}{4(k+1)}$
(i) $2x^2 + x - 3.$	$2(x + \frac{1}{4})^2 - \frac{25}{8}$		

2. By completing the square find the vertex of the following curves and state whether it is a maximum or minimum.

(a) $y = x^2 - 14x + 11.$	$(7, -38), \text{ min}$	(g) $y = 3x^2 - x + 1.$	$(\frac{1}{6}, \frac{11}{12}), \text{ min}$
(b) $y = x^2 + 5x.$	$(-\frac{5}{2}, -\frac{25}{4}), \text{ min}$	(h) $y = x^2 + kx.$	$(-\frac{k}{2}, -\frac{k^2}{4}), \text{ min}$
(c) $y = x^2 + 4x + 4.$	$(-2, 0), \text{ min}$	(i) $y = -2x^2 + ax - 3.$	$(\frac{a}{4}, \frac{a^2-24}{8}), \text{ max}$
(d) $y = x^2 - 3x + 8.$	$(\frac{3}{2}, \frac{23}{4}), \text{ min}$	(j) $y = 3x^2 + rx + mx - 1.$	$(-\frac{r+m}{6}, -\frac{(r+m)^2+12}{12}), \text{ min}$
(e) $y = 2x^2 + 12x - 5.$	$(-3, -23), \text{ min}$		
(f) $y = -x^2 + 2x + a.$	$(1, 1+a), \text{ max}$		

3. By completing the square find the line of symmetry of the following curves.

(a) $y = x^2 + 12x - 3.$	$x = -6$	(d) $y = 2x^2 - x + 5.$	$x = \frac{1}{4}$
(b) $y = x^2 - 7x.$	$x = \frac{7}{2}$	(e) $y = 3x^2 + 5x - 2.$	$x = -\frac{5}{6}$
(c) $y = -x^2 + 2x - 1.$	$x = 1$	(f) $y = -2x^2 + \frac{1}{a}x - 1.$	$x = \frac{1}{4a}$

4. Solve the following equations by completing the square.

(a) $x^2 + 4x - 3 = 0.$	$x = -2 \pm \sqrt{7}$	(c) $x^2 + x = 7.$	$x = \frac{-1 \pm \sqrt{29}}{2}$
(b) $x^2 = 2x + 5.$	$x = 1 \pm \sqrt{6}$	(d) $2x^2 - 8x - 1 = 0.$	$x = \frac{4 \pm 3\sqrt{2}}{2}$

5. Sketch the following curves by completing the square.

(a) $y = x^2 + 3x - 5.$	(b) $y = 2x^2 + x - 7.$
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6. Extending the concept. Find the minimum or maximum point(s) on the following graphs.

(a) $y = (x^2 - 1)^2 + 5.$

$(\pm 1, 5), \text{ min}$

(e) $y = (x^2 - px - qx + pq)^2 - pq.$

$(p, -pq) \text{ or } (q, -pq), \text{ min}$

(b) $y = (4x^2 - 9)^2 - 1.$

$(\pm \frac{3}{2}, -1), \text{ min}$

(c) $y = (x^2 + x - 6)^2 + k.$

$(2, k) \text{ or } (-3, k), \text{ min}$

(f) $y = x^4 - 4x^2 + 1.$

$(\pm \sqrt{2}, -2), \text{ min}$

(d) $y = -(3x^2 - 10x - 8)^2 + 5.$

$(4, 5) \text{ or } (-\frac{2}{3}, 5), \text{ max}$

(g) $y = -x^4 + 8x^2 - 9.$

$(\pm 2, 7), \text{ max}$

(h) $y = (x^2 - 2x + 2)^2 + 10.$

$(1, 11), \text{ min}$