

Quadratic Sequences

Last lesson we learnt how to find an expression for the n th term of a arithmetic (linear) sequence; i.e. one that goes up or down by a constant amount. We found

$$T = (\text{diff between terms})n + (\text{zeroth term}).$$

So for example the sequence $3, -1, -5, -9 \dots$ gives the formula $T = -4n + 7$. We spot arithmetic sequences by looking at the first difference and spotting the constant.

However, if the second difference is constant then you are dealing with a *quadratic sequence*. To work out the formula;

1. Halve the second difference to get the number of n^2 s (e.g. if you had a second difference of $+6$ you would have $+3n^2$).
2. Write out the original sequence above the terms of your number of n^2 s.
3. Subtract the n^2 s from the sequence to give the RESIDUE.
4. The residue will either be constant or a linear sequence. If it is a linear sequence then work out its formula.
5. Finally add the number of n^2 s to the formula for the residue and this will be the formula for the original sequence.

An Example

Find the formula for the sequence $3, 13, 27, 45, 67 \dots$

Difference table:

3	13	27	45	67
+10	+14	+18	+22	
	+4	+4	+4	

So we have quadratic sequence with $+2n^2$ (half of $+4$).

So write out sequence and $+2n^2$ and subtract to find residue.

SEQUENCE	3	13	27	45	67
$2n^2$	2	8	18	32	50
RESIDUE	1	5	9	13	17

So the formula for the residue is $4n - 3$ so the overall formula for the sequence $3, 13, 27, 45, 67 \dots$ is $T = 2n^2 + 4n - 3$.

Questions

1. Find the formula for the n th term of the following sequences:

(a) $3, 6, 11, 18, 27, 38, 51 \dots$

$n^2 + 2$

(b) $-19, -15, -9, -1, 9, 21, 35 \dots$

$n^2 + n - 21$

(c) $4, 10, 20, 34, 52, 74, 100 \dots$

$2n^2 + 2$

(d) $2, 9, 22, 41, 66, 97, 134 \dots$

$3n^2 - 2n + 1$

(e) $2, 12, 26, 44, 66, 92, 122 \dots$

$2n^2 + 4n - 4$

(f) 14, 67, 122, 179, 238, 299, 362...

$$n^2 + 50n - 37$$

2. Use the formulae for the above expressions to work out

- (a) the 50th term of the sequence,
- (b) the 1000th term of the sequence.

3. Now work out an expression for the n th term of the following mixture of arithmetic and linear sequences:

(a) 3, 3.5, 4, 4.5, 5, 5.5, 6...

$$T = 0.5n + 2.5$$

(b) 3, 9, 17, 27, 39, 53, 69...

$$T = n^2 + 3n - 1$$

(c) 13, 6, -1, -8, -15, -22, -29...

$$T = -7n + 20$$

(d) -2, 4, 14, 28, 46, 68, 94...

$$T = 2n^2 - 4$$

(e) -8.5, -10, -11.5, -13, -14.5, -16, -17.5...

$$T = -1.5n - 7$$

(f) 1, 5, 15, 31, 53, 81, 115,...

$$T = 3n^2 - 5n + 3$$

(g) 21.1, 22.2, 23.3, 24.4, 25.5, 26.6, 27.7...

$$T = 1.1n + 20$$

(h) -0.6, 2.8, 6.2, 9.6, 13, 16.4, 19.8...

$$T = 3.4n - 4$$