4th Year "Beginning of Year Test" Revision Sheet

This document attempts to sum up what you will need for the upcoming test next week. Email me on `jonathan.m.stone@gmail.com` with any queries.

Equations

- With simple equations we must get the x's to one side and the numbers to the other. We must do this by doing the *same thing to both sides*. For example:

$$\begin{aligned} 2x + 15 &= 5x - 3 \\ 15 &= 3x - 3 && \text{subtracting } 2x, \\ 18 &= 3x && \text{adding } 3, \\ x &= 6 && \text{dividing by } 3. \end{aligned}$$

- When equations have denominators (e.g. $\frac{q+3}{5} - \frac{2q-3}{4} = \frac{q+1}{2} + 2$) we can get rid of them by multiplying by the entire equation by the LCM of the denominators. So in this example there are 4 terms in the equation so I must make sure I multiply *all* 4 of them by 20. The denominators will then *all* cancel with what you are multiplying by.

$$\begin{aligned} \frac{q+3}{5} - \frac{2q-3}{4} &= \frac{q+1}{2} + 2 \\ 20 \times \frac{(q+3)}{5} - 20 \times \frac{(2q-3)}{4} &= 20 \times \frac{(q+1)}{2} + 20 \times 2 \\ 4(q+3) - 5(2q-3) &= 10(q+1) + 40 \\ 4q + 12 - 10q + 15 &= 10q + 10 + 40 \\ -16q &= 23 \\ q &= -1\tfrac{7}{16}. \end{aligned}$$

- When equations have the unknown in the denominator we must multiply by the denominator. For example

$$\begin{aligned} \frac{20}{2x+1} &= 4 \\ (2x+1) \times \frac{20}{2x+1} &= 4 \times (2x+1) \\ 20 &= 8x + 4 \\ x &= 2. \end{aligned}$$

- For equations with words we always define a variable (or variables). Then proceed to write to write down an equation and solve it. For example; "I think of a number. I add three to it. I then multiply the result by 2 and then add one. Finally I divide by 7. The results is 3. What is the number?" Let $n =$ "the number though of". The equation can be written as

$$\frac{2(n+3)+1}{7} = 3$$

  which solves to $n = 7$.

- For simultaneous equations usually it is best to use substitution. If there is an x or a y on its own, you isolate it and put it into the other equation. For example solve

$$\begin{aligned} 2x + 3y &= 7 \\ 3x + y &= 7 \end{aligned}.$$

From the second we find $y = 7 - 3x$. Put this into first equation to get $2x + 3(7 - 3x) = 7 \quad \Rightarrow \quad -7x = -14 \quad \Rightarrow \quad \boxed{x = 2} \quad \Rightarrow \quad \boxed{y = 1}$.

- If there is not an x or y on its own then you must make the x's or y's the same and then substitute *that* into the other equation. For example in

$$\begin{aligned} 2x + 3y &= 7 \\ 3x - 7y &= 2 \end{aligned}$$

you would multiply the top one by 3 and the bottom one by 2 to obtain

$$\begin{aligned} 6x + 9y &= 21 \\ 6x - 14y &= 4 \end{aligned}.$$

Then isolate the $6x$ from one of the equations and substitute into the other one like in the simpler case. In this case you should find $x = 2\frac{9}{23}$ and $y = \frac{17}{23}$.

Linear Graphs

- To draw a straight line graph from an equation we can draw a table of x and y. For example draw the line $y = -2x - 4$ we fill in the table

x	0	2	-3
y	-4	-8	2

We would then plot the points $(0, -4)$, $(2, -8)$ and $(-3, 2)$ and join them up with a ruler to find the line.

- However, a faster way is to look at the line in the form $y = mx + c$ where m is the gradient and c is the y-axis intercept. So the line $y = 4x - 2$ has intercept -2 and gradient 4. So it goes through the point $(0, -2)$. From this we go across *one to the right* and *up four* to find our next point (and so on).

- If the line is not given to you with y as the subject then you could make it the subject. For example

$$4x + 3y = 6 \qquad \Rightarrow \qquad y = -\frac{4}{3}x + 2.$$

So we see the gradient is $-\frac{4}{3}$ and the y-intercept is 2. From $(0, 2)$ we go across right 3 and down 4 to locate the next point.

- The gradient between two points is defined to be the difference in y divided by difference in x. Therefore between (x_1, y_1) and (x_2, y_2) it is

$$\text{Gradient} = \frac{\text{Difference in } y}{\text{Difference in } x} = \frac{y_2 - y_1}{x_2 - x_1}.$$

- To find the equation of line from a point and a gradient you substitute the point into the equation $y = mx + c$ and solve for c. For example find the equation of the line of gradient 3 through the point $(12, -5)$. Well $y = 3x + c$ so placing $(12, -5)$ in we find

$$\begin{aligned} y &= 3x + c \\ -5 &= 3 \times 12 + c \\ -41 &= c. \end{aligned}$$

So the line is $y = 3x - 41$.

- To find the equation of a line from two points we find the gradient between the points and then follow the same procedure as above. For example find the equation of the line through $(3, -4)$ and $(6, 7)$. The gradient is $\frac{7-(-4)}{6-3} = \frac{11}{3}$. Therefore the line is $y = \frac{11}{3}x + c$ through both the points. You then substitute in one of the points and find c. Therefore

$$\begin{aligned} y &= \tfrac{11}{3}x + c \\ 7 &= \tfrac{11}{3} \times 6 + c \\ -15 &= c. \end{aligned}$$

  Therefore the line is $y = \frac{11}{3}x - 15$. This can also (more elegantly) be multiplied by 3 to yield $3y - 11x + 45 = 0$.

Trigonometry

- In trigonometry you will always be referring to *right angled* triangles. (If there isn't one then you will need to construct one somehow.) There will always be two sides and one angle that you are 'interested in'. You will know two and you will need to work out the third. Assign the unknown length or angle a letter (usually a for angle and x for length).

- All you need to remember is SohCahToa, which is a way of remembering that

$$\sin a = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos a = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan a = \frac{\text{opposite}}{\text{adjacent}}.$$

  The hypotenuse is fixed, and the adjacent and opposite lengths are relative to the angle you are 'interested in'.

- Then write down the equation and solve for the unknown (it will always be sin, cos or tan of an angle equals a length over a length). Here are three initial equations and their solutions; make sure you can do all of them.

$$\begin{aligned} \sin a &= \frac{4}{5} & \cos 40 &= \frac{x}{3} & \tan 70 &= \frac{15}{x} \\ a &= \sin^{-1}\left(\frac{4}{5}\right) & x &= 3\cos 40 & x &= \frac{15}{\tan 70} \\ a &= 53.1 \text{ (3sf)} & x &= 2.30 \text{ (3sf)} & x &= 5.46 \text{ (3sf)} \end{aligned}$$

- The rule of thumb is that when the angle is unknown you will need to use the inverse function. For example if $\tan a = \frac{3}{7}$ then $a = \tan^{-1}\left(\frac{3}{7}\right)$

Percentages & Ratios

- For simple questions it is important to think of a scale factor. For example to increase a quantity by 5% we multiply by 1.05. Do decrease a quantity by 7% we multiply by 0.93.

- *All* percentage questions are most easily solved by making a table of percentage and amount and then cross multiplying and solving for the unknown. Remember that the original amount (the quantity before any change) is always the 100% amount.

  For example, I bought a car 3 years ago and it has since lost 41% of its value. It is now valued at £11800. What was its original value?

Amount	Percentage
x	100
11800	59

So $59x = 1180000$, therefore $x = £20000$.

- Compound interest is most easily summed up by the formula $A = P \times (1 + \frac{r}{100})^n$, where P is the original amount invested, A is the total amount including new interest, r is the interest rate and n is the number of years invested. For example, I invested an amount of money 7 years ago at 4.5% compound interest and there is now £4082.59 in my account. How much did I invest? Put the numbers into the formula and solve;

$$4082.59 = P \times 1.045^7 \qquad \text{so} \qquad P = \frac{4082.59}{1.045^7} = £3000.$$

For revision, make sure you can do the following questions. If not, then you need to speak to me, or go into Rayner, find the relevant section and do lots of questions.

1. **Equations**

   Solve

   (a) $2x + 3 = 7x - 5$ $\boxed{x = \frac{8}{5}}$

   (b) $2(2x + 4) - 3(x - 1) = 7$ $\boxed{x = -4}$

   (c) $4 - (x - 2) = 6$ $\boxed{x = 0}$

   (d) $\dfrac{x+3}{2} + \dfrac{2x+1}{5} = x$ $\boxed{x = 17}$

   (e) $7 = \dfrac{2x-1}{3} - \dfrac{x-4}{4}$ $\boxed{x = 15\frac{1}{5}}$

   (f) $5 = \dfrac{3}{2x+1}$ $\boxed{x = -\frac{1}{5}}$

   (g) $\dfrac{6}{x+1} - 4 = \dfrac{5}{x+1}$ $\boxed{x = -\frac{3}{4}}$

   (h) I think of a number. I divide it by 11, then I add 2, then I divide the result by 4. The result is one more than the number I thought of. What was the number? $\boxed{n = 11}$

   (i) $\begin{pmatrix} 2x + y = 4 \\ 3x + 7y = 17 \end{pmatrix}$ $\boxed{x = 1,\ y = 2}$

   (j) $\begin{pmatrix} 4x - 3y = 2 \\ 3x + 2y = -7 \end{pmatrix}$ $\boxed{x = -1,\ y = -2}$

2. **Line graphs**

   (a) Draw the line $y = \frac{2}{3}x + 1$ quickly.

   (b) Write the line $2x - 3y + 7 = 0$ in the form $y = mx + c$. $\boxed{y = \frac{2}{3}x + \frac{7}{3}}$

   (c) Find the equation of the line with gradient $\frac{1}{3}$ through the point $(4, -1)$. $\boxed{y = \frac{1}{3}x - \frac{7}{3}}$

   (d) Find the equation of the line through $(3, 5)$ and $(5, -2)$. $\boxed{y = -\frac{7}{2}x + \frac{31}{2}}$

   (e) Write the line $y = -\frac{1}{3}x - \frac{7}{3}$ in the form $ax + by = c$ where a, b and c are integers. $\boxed{x + 3y = -7}$

3. **Trigonometry**

   For the following questions triangle ABC has a right angle at B.

   (a) If the angle at A is 20 degrees and $AC = 8$, find BC. $\boxed{2.736 \text{ (4sf)}}$

   (b) If the angle at A is 55 degrees and $BC = 10$, find AB. $\boxed{7.002 \text{ (4sf)}}$

   (c) If $AB = 7$ and $BC = 8$, find the angle at A. $\boxed{48.81}$

4. **Percentages**

   (a) The sale price of a T-Shirt after it has been reduced by 15% is £17. What was the original price of the shirt? $\boxed{£20}$

   (b) I invest £4,000 at 7% compound interest for 14 years. How much do I have in my account now? $\boxed{£10,314.14}$