

## Inductive Sequences (& A Sprinkling Of Algebraic Fractions)

Remember that inductively defined sequences are not as scary as they look. Just remember to read  $u_{n+1} = 2u_n + 3$  as “The next number in the sequence is twice the previous term plus three”. You will also need a starting value like  $u_1 = 4$ . Therefore this example would yield 4, 11, 25, 53, 109, ...

1. Find the first five terms in the following inductively defined sequences.

(a)  $u_{n+1} = 3u_n - 1$  with  $u_1 = 7$ .

7, 20, 59, 176, 527

(b)  $u_{n+1} = 2 - u_n$  with  $u_1 = 2$ .

2, 0, 2, 0, 2

(c)  $u_{n+1} = \frac{1}{u_n}$  with  $u_1 = 4$ .

4,  $\frac{1}{4}$ , 4,  $\frac{1}{4}$ , 4

(d)  $u_{n+1} = u_n + 1$  with  $u_1 = k$ .

$k, k + 1, k + 2, k + 3, k + 4$

(e)  $u_{n+1} = 2 + \frac{3}{u_n}$  with  $u_1 = 1$ .

1, 5,  $\frac{13}{5}$ ,  $\frac{41}{13}$ ,  $\frac{121}{41}$

(f)  $u_{n+1} = 2^{u_n} - 1$  with  $u_1 = 2$ .

2, 3, 7, 127,  $1.70 \times 10^{38}$

(g)  $u_{n+1} = 3^{u_n} - 2^{u_n}$  with  $u_1 = 2$ .

2, 5, 211, ...

Now with the next term being dependent on the previous two terms...

(h)  $u_{n+1} = u_n + 2u_{n-1}$  with  $u_1 = 1$  and  $u_2 = 2$ .

1, 2, 4, 8, 16

(i)  $u_{n+1} = \frac{1}{u_n} + \frac{1}{u_{n-1}}$  with  $u_1 = -1$  and  $u_2 = 3$ .

□

(j)  $u_{n+1} = \frac{1}{u_n + u_{n-1}}$  with  $u_1 = -1$  and  $u_2 = 3$ .

□

(k)  $u_{n+1} = u_n + u_{n-1}$  with  $u_1 = a$  and  $u_2 = a^2$ .

□

2. Find the specified term in each sequence (fully simplified, of course).

(a)  $u_{n+1} = \frac{2}{u_n} + 1$  with  $u_1 = 3$ , find  $u_4$ .

$\frac{21}{11}$

(b)  $u_{n+1} = \frac{2}{u_n} + 1$  with  $u_1 = x$ , find  $u_4$ .

$\frac{5x+6}{3x+2}$

(c)  $u_{n+1} = \frac{1}{u_n+1}$  with  $u_1 = 3$ , find  $u_4$ .

$\frac{5}{9}$

(d)  $u_{n+1} = \frac{2}{u_n+3}$  with  $u_1 = x$ , find  $u_4$ .

$\frac{6x+22}{11x+39}$

(e)  $u_{n+1} = \frac{2+u_n}{3+2u_n}$  with  $u_1 = x$ , find  $u_4$ .

$\frac{21x+34}{34x+55}$

(f)  $u_{n+2} = \frac{1}{u_{n+1}+u_n}$  with  $u_1 = x$  and  $u_2 = y$ , find  $u_5$ .

$\frac{y^3+2xy^2+x^2y+y+x}{2y^2+3xy+x^2+1}$