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## STAGE 2 INTERMEDIATE REVISION SHEET

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This document attempts to sum up the contents of the Intermediate Tier Stage 2 Module.

There are two exams, each 25 minutes long. One allows use of a calculator and the other doesn't. Together they represent 15% of the GCSE.

Before you go into the exam make sure you are fully equipped with two pens, two pencils, a calculator, a ruler, a protractor and a pair of compasses. Also be sure not to panic; it is not uncommon to get stuck on a question (I've been there!). Just continue with what you can do and return at the end to the question(s) you have found hard. If you have time check all your work, especially the first question you attempted... always an area prone to error.

I am always available on [jonathan.m.stone@gmail.com](mailto:jonathan.m.stone@gmail.com) to answer any questions you may have. Please do not hesitate.

*JMS*

### Working With Number

- You must be very fluent in rounding numbers to both decimal places and significant figures. Best done by a few examples.

ORIGINAL NUMBER	1SF	2SF	1DP	2DP
2.459	2	2.5	2.5	2.46
1256.897	1000	1300	1256.9	1256.90
0.005741	0.006	0.0057	0.0	0.01
2.4999	2	2.5	2.5	2.50

- When *estimating* you normally round all the components of a calculation to 1 significant figure and carry out the calculation. For example estimate  $\frac{245 \times 26}{17.8}$ . This becomes roughly  $\frac{200 \times 30}{20} = 300$ .
- Ratios are usually canceled down to their lowest form in integers. For example 5 : 15 : 20 goes to 1 : 3 : 4 and  $2 : \frac{1}{3}$  goes to 6 : 1.
- In question where they say  $3 : 4 = x : 16$  it is usually pretty obvious to see what to do. In this case  $x = 12$  because the right hand is four times the left hand.
- When dividing quantities in a ratio you sum the components of the ratio and divide into the quantity. You then multiply the original ratio by this number. For example divide £154 in the ratio 3 : 4 : 4.

$$3 + 4 + 4 = 11 \quad \Rightarrow \quad \frac{154}{11} = 14 \quad \Rightarrow \quad (3 \times 14) : (4 \times 14) : (4 \times 14) = \text{£}42 : \text{£}56 : \text{£}56.$$

### Indices & Calculators

- When working out formulae for certain values you must make sure to use BODMAS. For example when calculating  $4n^2 + n$  you must do the  $n^2$  first and then multiply by 4 and then add  $n$ . So  $4n^2 + n$  when  $n = 3$  is  $4 \times 3^2 + 3 = 39$ .

- Know that  $7^4$  means  $7 \times 7 \times 7 \times 7$ . When putting  $7^4$  on a calculator you need to type  $7 \square \wedge \square 4$ . Similarly for  $\sqrt[5]{34}$  you type  $5 \square \sqrt[y]{x} \square 34$ .
- You must be doubly careful when inserting negative numbers into expressions. For example what is the value of  $3a^2 - 4b$  when  $a = -3$  and  $b = -2$ ? Answer;  $3 \times (-3)^2 - 4 \times (-2) = 27 - (-8) = 35$ .
- Make sure you read the separate handout on trial and improvement method for solving equations.
- All numbers (both very big and very small) can be written in *standard form*. A number in standard form is always a number **between one and ten**, times ten to a power (i.e.  $A \times 10^n$ ). Here are a few examples...make sure you can convert either way! The power is found by how many times you move the decimal place and in which direction.

ORDINARY FORM	STANDARD FORM
360	$3.6 \times 10^2$
400000	$4 \times 10^5$
8070	$8.07 \times 10^3$
0.00345	$3.45 \times 10^{-3}$
0.45	$4.5 \times 10^{-1}$

- On your calculator the sum  $(3.2 \times 10^6) + (4.6 \times 10^5)$  can be entered as  $(3.2 \square \text{EXP} \square 6) + (4.6 \square \text{EXP} \square 5)$  and you should find the answer is 3660000 which is  $3.66 \times 10^6$  in standard form.

## Graphs

- To draw a straight line graph from an equation, always draw a table of  $x$  and  $y$ . For example draw the line  $y = -2x - 4$  we fill in the table;

$x$	0	2	-3
$y$	-4	-8	2

We would then plot the points  $(0, -4)$ ,  $(2, -8)$  and  $(-3, 2)$  and join them up with a ruler to find the line.

- If the line is not given to you with  $y$  as the subject then make it the subject. For example

$$4x + 3y = 6 \quad \Rightarrow \quad y = \frac{6 - 4x}{3}.$$

Then do the  $xy$  table as above.

- Know how to read from conversion graphs and real world graphs (see pages 22 & 23). Know that on distance–time graphs you are often asked for speeds (=dist/time).
- To graph quadratic functions you need to draw a detailed table, devoting a column for each section of the equation. For example to draw  $y = 2x^2 - 2x - 4$  make the following table:

$x$	$2x^2$	$-2x$	$-4$	$y$
-3	18	6	-4	20
-2	8	4	-4	8
-1	2	2	-4	0
0	0	0	-4	-4
1	2	-2	-4	-4
2	8	-4	-4	0
3	18	-6	-4	8

And then plot the points  $(-3, 20)$ ,  $(-2, 8)$ ,  $\dots$ ,  $(3, 8)$  and join with a smooth curve. You should find that it is a U-shaped curve crossing the  $x$ -axis at  $-1$  and  $2$ .

- If the number of  $x^2$ 's is positive (e.g.  $y = 2x^2 + \dots$ ) then the graph will be U-shaped and if the number of  $x^2$ 's is negative (e.g.  $y = -3x^2 + \dots$ ) then the graph will be  $\cap$ -shaped.

### Working With Algebra

- When multiplying out something like  $(x+3)(x-5)$  use the 'crescent technique'. Therefore

$$(x+3)(x-5) = x^2 - 5x + 3x - 15 = x^2 - 2x - 15.$$

Another example;  $(2a+b)(3a-2b) = 6a^2 - 4ab + 3ab - 2b^2 = 6a^2 - ab - 2b^2$ . And another(!);  $(2x-y)^2 = (2x-y)(2x-y) = 4x^2 - 2xy - 2xy + y^2 = 4x^2 - 4xy + y^2$ .

- When factorising a quadratic expression you need to find two numbers that multiply ( $\times$ ) to the constant and add ( $+$ ) to the number of  $x$ 's. For example in  $x^2 + 11x - 12$  we need to find two numbers that  $\times$  to  $-12$  and  $+$  to  $11$ . Clearly they are  $12$  and  $-1$ , so  $x^2 + 11x - 12 = (x+12)(x-1)$ .
- When factorising a quadratic expression such as  $3x^2 - 9x$  you should notice that there is no constant term. We therefore just factorise it like normal;  $3x^2 - 9x = 3\underline{x}x - 9\underline{x} = 3x(x-3)$ .
- When changing the subject of a formula/equation you must make sure (as when solving equations) that you do the same thing to both sides. For example make  $b$  the subject in  $A = \frac{1}{2}(a+b)h$ .

$$A = \frac{1}{2}(a+b)h \quad \Rightarrow \quad \frac{2A}{h} = a+b \quad \Rightarrow \quad b = \frac{2A}{h} - a$$

- To solve inequalities you just treat them (almost) like equations; ie doing the same to both sides. The only thing you need to be careful about is multiplying or dividing by negative numbers (if you do, then you 'flip' the sign of the inequality). For example solve  $3x - 5 > x + 7$ :

$$\begin{aligned} 3x - 5 &> x + 7 \\ 2x - 5 &> 7 && \text{subtract } x, \\ 2x &> 12 && \text{add } 5, \\ x &> 6 && \text{divide by } 2. \end{aligned}$$

So the solution is all values of  $x$  greater than  $6$ .

- 'Triple' inequalities represent a region of the number line. For triple inequalities you must do the same to *all three* components and leave  $x$  in the middle. For example find all integer solutions to  $-3 < 4x - 7 \leq 5$ :

$$\begin{aligned} -3 < 4x - 7 &\leq 5 \\ 4 < 4x &\leq 12 && \text{add } 7, \\ 1 < x &\leq 3 && \text{divide by } 4. \end{aligned}$$

Therefore the only integer solutions are  $x = 2$  and  $x = 3$  as the inequality does not include  $1$ .

## Pythagoras & Trigonometry

- Pythagoras' Theorem states that in a *right angled* triangle with sides  $a$ ,  $b$  and  $c$  where  $c$  is the hypotenuse (longest length; always opposite the right angle) then  $a^2 + b^2 = c^2$ . In other words "the short one squared plus the short one squared equals the long one squared".
- You will always know two of the sides and not know the other. Assign a letter to the unknown side and then write down an equation for the triangle. Solve for the unknown and *don't forget to square root at the end!* For example you might get

$$x^2 + 5^2 = 9^2 \quad \Rightarrow \quad x^2 = 81 - 25 \quad \Rightarrow \quad x = \sqrt{56}.$$

- In trigonometry you will always be referring to *right angled* triangles. There will always be two sides and one angle that you are 'interested in'. You will know two and you will need to work out the third. Assign the unknown length or angle a letter (usually  $a$  for angle and  $x$  for length).
- All you need to remember is SOHCAHTOA, which is a way of remembering that

$$\sin a = \frac{\text{opposite}}{\text{hypotenuse}} \quad \cos a = \frac{\text{adjacent}}{\text{hypotenuse}} \quad \tan a = \frac{\text{opposite}}{\text{adjacent}}.$$

The hypotenuse is fixed, and the adjacent and opposite lengths are relative to the angle you are 'interested in'.

- Then write down the equation and solve for the unknown (it will always be sin, cos or tan of an angle equals a length over a length). Here are three initial equations and their solutions; make sure you can do all of them.

$$\begin{array}{lll} \sin a = \frac{4}{5} & \cos 40 = \frac{x}{3} & \tan 70 = \frac{15}{x} \\ a = \sin^{-1}\left(\frac{4}{5}\right) & x = 3 \cos 40 & x = \frac{15}{\tan 70} \\ a = 53.1 \text{ (3sf)} & x = 2.30 \text{ (3sf)} & x = 5.46 \text{ (3sf)} \end{array}$$

- The rule of thumb is that when the angle is unknown you will need to use the inverse function. For example if  $\tan a = \frac{3}{7}$  then  $a = \tan^{-1}\left(\frac{3}{7}\right)$
- The angle of elevation and angle of depression are shown clearly on page 62.

## Properties Of Circles

- The area of a circle is given by  $A = \pi r^2$ . Make sure you are using the radius in this formula and *not* the diameter.
- The circumference of a circle is given by  $C = 2\pi r$ .
- When you have a compound shape (for example at the top left of page 68) you must cut it up into bits of shapes you know. In the example you can see that it can be cut up into a rectangle 100 by 60 and two half circles of radius 30. Therefore

$$\text{Area} = (100 \times 60) + (\pi \times 30^2) \quad \text{and} \quad \text{Perimeter} = 200 + (2 \times \pi \times 30).$$

- The tangent to a circle and the radius at that point are *always* at right angles. See page 68.

- Tangents drawn from a point to both sides of a circle are equal in length. This implies that there will exist a kite with two right angles in it. Cut it in two to form a right angled triangle. See page 68.
- Questions involving tangents to circles are found *very* hard by GCSE students. Keep your head about you and don't try to get the answer in one go; just work out what you can and hopefully the answer will just 'drop out' and if it doesn't you may still get marks for your working.

## Translations, Enlargements & Scale Drawings

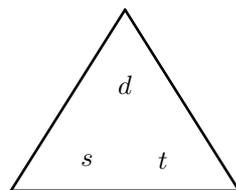
- In any **transformation** in this section the 'object' is the original shape and the 'image' is the object after transformation.
- A **translation** is a sliding movement in the  $xy$ -plane without rotation, reflection or enlargement. It is usually described by a column vector of the form  $\begin{pmatrix} \text{movement right} \\ \text{movement up} \end{pmatrix}$ . So the vector  $\begin{pmatrix} 4 \\ -3 \end{pmatrix}$  is a translation 4 right and 3 down.
- To translate an object by a given column vector (e.g.  $\begin{pmatrix} -3 \\ 5 \end{pmatrix}$ ) it is best to consider *one* point on the shape and move it by the vector (3 left and 5 up in this case) and then redraw the rest of the shape from that point.
- If you are given two shapes and it has been translated, then to describe the translation we consider the same point on both shapes and see how it has moved. We then say that the transformation is a translation and state the column vector.
- An **enlargement** changes the size (but not the shape) of an object. The image of an enlargement is said to be *similar* to the object. The *scale factor* describes how many times larger the shape has become. For example if the scale factor is three then *all* the lengths on the object are three times longer in the image.
- There is also the centre of enlargement to consider. If you are given two similar shapes and asked to find the centre of the enlargement then you just 'ray trace' from common points on object and image. Where these lines cross is the centre of enlargement. See bottom of page 72.
- You must also know how to enlarge a shape with a given scale factor and centre. You take a single point on the image and work out the column vector *from* the centre *to* the point (e.g.  $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ ). You then multiply the vector by the scale factor (e.g. for a scale factor of two;  $2 \times \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$ ). The image point will then be  $\begin{pmatrix} 4 \\ 6 \end{pmatrix}$  from the centre. Given that the scale factor is two, you then redraw the whole shape from that point but with all lengths twice as long.
- When asked to describe an enlargement you must include both the centre of enlargement and the scale factor (and also, of course, that it is an enlargement!).
- In a **scale drawing** the *scale* is a ratio showing how real life relates to the scale drawing. For example 1 : 25000 tells us 1cm in the map is 25000cm on the ground. Therefore 1cm = 250m and 4cm = 1km.

## Mensuration

- Whenever a figure is given to the nearest unit then there is a possible error of half a unit either side. For example if the temperature is  $T = 23$  degrees centigrade to the nearest degree, then  $22.5 \leq T < 23.5$ . The *maximum error* is 0.5 degrees either side and  $22.5$

is the *lower bound* (smallest possible value) and 23.5 is the *upper bound* (largest possible value).

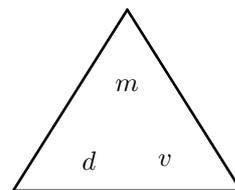
- The area of a triangle is  $A = \frac{1}{2}bh$  where  $h$  refers to the perpendicular height. When working out the areas of complex shapes remember to cut it up into smaller, simpler shapes and work them out bit by bit.
- The volume of a *cuboid* is obvious; Volume = length  $\times$  base  $\times$  height =  $lbh$ .
- The volume of a prism is the (area of the *extended* face)  $\times$  (its length). It is almost *always* best to calculate the area of the face first (no matter how fiddly) and *then* multiply by the length. The surface area is then done on an *ad hoc*<sup>1</sup> basis.
- Cylinders are prisms with circular cross section. Vol =  $\pi r^2 h$  and S.A. =  $2\pi r^2 + 2\pi r h$ . In S.A. the  $2\pi r^2$  is the area of the circular top and bottom; the  $2\pi r h$  is the area of the ‘curved face’.<sup>2</sup>
- When faced with a calculation in mensuration you *must* be careful about units. To calculate an area or volume you will need to convert all lengths to one consistent unit (e.g. mm, cm, m, km, etc).
- You must be familiar with the equations connecting distance, speed and time. Also you must know the equations connecting mass, density and volume. Most easily remembered by the below triangles. Cover up the one you want to find the equation. “My Dirty Van” is a way of remembering the second.



$$d = s \times t$$

$$s = d/t$$

$$t = d/s$$



$$m = d \times v$$

$$d = m/v$$

$$v = m/d$$

## Probability & Scatter Graphs

- With probability questions it is best to practice! Therefore do the questions on page 89/90.
- If  $A$  and  $B$  are both mutually exclusive (i.e. both can't happen together; for example rolling a 5 and a 6 with a dice) then  $P(A \text{ or } B) = P(A) + P(B)$ . For example probability of a five or a six when rolling a dice is  $P(5 \text{ or } 6) = P(5) + P(6) = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$ .
- If the probability of an event happening is  $p$ , then the probability of it not happening is  $1 - p$ . For example if the probability of Tom getting a grade B is 0.34, then the probability of him not getting a B is  $1 - 0.34 = 0.66$ . :-)
- If I flick a biased coin 50 times and get 40 heads and 10 tails then my *estimated probability* of a head is  $\frac{40}{50} = \frac{4}{5}$ . Another example if I play 500 games of chess and I win 300, then my estimated probability of winning is  $\frac{300}{500} = \frac{3}{5}$ . This is sometimes called the *relative frequency*. The estimated probability should become more accurate the more trials I do.
- Scatter graphs can be used to see how two sets of data are related. If the points are almost in a line then there is a high degree of *correlation* between the variables. We would expect high correlation between people's times for the 100m sprint and 200m sprint (fat people are slow at both etc.)

<sup>1</sup>*ad hoc* = Fly by the seat of your pants.

<sup>2</sup>Curved Face = The bit where the sticker goes on a can of beans.

- There is *positive correlation* if, as one variable increases, so does the other. There is *negative correlation* if, as one increases, the other decreases. You can usually draw the *line of best fit*. You can use this to estimate values from the graph. The line of best fit does **not** always go through the origin (0, 0).

## Handling Data

- To be honest the first section of this chapter is such a nightmare to write out and we did it all in class so it's all in your notes. Please look at page 117 & 118 for summary of the key points. You must be good with the following:
  - Drawing and reading pie charts. Page 96.
  - Drawing and reading line graphs. Page 97.
  - Drawing a histogram from a grouped data set. Page 98.
  - A frequency polygon is the line formed by joining up the mid-points of the tops of a histogram. Page 99.
- Given raw data you must be able to construct a stem and leaf diagram. Remember to define the scale (e.g. 4|5 means 0.45). Page 99.
- Given a frequency distribution you create a new column (the product of the first two;  $xf$ ) and sum it to help you work out the *mean*. For example

$x$	$f$	⇒	Create	⇒	$x$	$f$	$xf$
0	1	⇒	Extra	⇒	0	1	0
1	3	⇒	Column	⇒	1	3	3
2	5	⇒	&	⇒	2	5	10
3	4	⇒	Sum	⇒	3	4	12
4	2	⇒		⇒	4	2	8
	15					15	33

So the mean is  $33/15$ .

- If you have grouped data ( $0 \leq x < 5$ ,  $5 \leq x < 10$ , etc.) then the best you can do is *estimate the mean* by using the mid-point (2.5, 7.5, etc.) and carry out the same process as above. Page 105.
- Find the *median* of a set of data. The median occurs at the  $(\frac{n+1}{2})$ th data point. For example, median of 2, 3, 4 is 3, but the median of 1, 2, 3, 4 is 2.5 (the average of 2 & 3). For grouped data you can only state the class interval that contains the median (e.g. it lies in the class  $15 \leq x < 20$ ).
- The *lower quartile* is the piece of data one quarter of the way in to a set of data. The *upper quartile* is the piece of data three quarters of the way in. The *interquartile range* is defined as the upper quartile – lower quartile. It is a measure of **spread** (how spread out the data is).
- The cumulative frequency is a “running total” of the frequencies as you go up the values. For example

$x$	$f$	⇒	Create	⇒	$x$ (upper limit of)	cum. freq
$0 \leq x < 5$	8	⇒	Cumulative	⇒	5	8
$5 \leq x < 10$	13	⇒	Frequency	⇒	10	21
$10 \leq x < 15$	17				15	38
$15 \leq x < 20$	10				20	48

Plot the second of these tables and join it with a smooth curve to form the *cumulative frequency curve*. From this the median and the two quartiles can be found. Page 107/8.

- Once these values are found we can draw a *box and whisker diagram*. The box and whisker diagram uses five values: the minimum, the maximum, the lower quartile, the upper quartile and the median. It is good for showing spread and comparing two quantities. This is clearly explained at the bottom of page 108.