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## STAGE 1 HIGHER REVISION SHEET

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This document attempts to sum up the contents of the Higher Tier Stage 1 Module.

There are two exams, each 25 minutes long. One allows use of a calculator and the other doesn't. Together they represent 15% of the GCSE.

Before you go into the exam make sure you are fully equipped with two pens, two pencils, a calculator, a ruler, a protractor and a pair of compasses. Also be sure not to panic; it is not uncommon to get stuck on a question (I've been there!). Just continue with what you can do and return at the end to the question(s) you have found hard. If you have time check all your work, especially the first question you attempted... always an area prone to error.

I am always available on [jonathan.m.stone@gmail.com](mailto:jonathan.m.stone@gmail.com) to answer any questions you may have. Please do not hesitate.

*JMS*

### Standard Form, Powers, Fractions & Decimals

- The factors of an integer (whole number) are the integers which divide into it without remainder. For example the factors of 8 are 1, 2, 4, 8. If a number has exactly two distinct factors (itself and one) then it is *prime*. The first few prime numbers are therefore 2, 3, 5, 7, 11, 13, 17, ... Note that one is *not* prime!
- You must be able to write any integer (whole number) as a *product of its prime factors*. For example  $630 = 2 \times 3^2 \times 5 \times 7$ .
- The highest common factor (HCF) of two (or more) numbers is the largest number that divides into them all. The lowest common multiple (LCM) of two (or more) numbers is the smallest number into which they all divide.
- To find the HCF or LCM of two (or more numbers) we write the numbers out in their prime factor form (written in full with  $3^2$  written as  $3 \times 3$  etc.). The HCF is the product of the numbers (counted once only, each) of numbers common to all of them. To find the LCM we write out one of the numbers and then multiply in the components we have not already dealt with from the other numbers.
- For example find the HCF and LCM of 1260, 924 and 1960. First we write in prime factor form thus:

$$1260 = \boxed{2} \times \boxed{2} \times 3 \times 3 \times 5 \times \boxed{7}$$

$$924 = \boxed{2} \times \boxed{2} \times 3 \times \boxed{7} \times 11$$

$$1960 = \boxed{2} \times \boxed{2} \times 2 \times 5 \times \boxed{7} \times 7$$

So HCF =  $2 \times 2 \times 7 = 28$  and LCM =  $(2 \times 2 \times 3 \times 3 \times 5 \times 7) \times (11) \times (2 \times 7) = 194040$ .

- All numbers (both very big and very small) can be written in *standard form*. A number in standard form is always a number **between one and ten**, times ten to a power (i.e.  $A \times 10^n$ ). Here are a few examples... make sure you can convert either way! The power is found by how many times you move the decimal place and in which direction.

ORDINARY FORM	STANDARD FORM
360	$3.6 \times 10^2$
400000	$4 \times 10^5$
8070	$8.07 \times 10^3$
0.00345	$3.45 \times 10^{-3}$
0.45	$4.5 \times 10^{-1}$

- On your calculator the sum  $(3.2 \times 10^6) + (4.6 \times 10^5)$  can be entered as  $(3.2 \text{ [EXP] } 6) + (4.6 \text{ [EXP] } 5)$  and you should find the answer is 3660000 which is  $3.66 \times 10^6$  in standard form.
- Notation:  $0.\dot{3} = 0.3333333\dots$  and  $0.12\dot{3}4\dot{5} = 0.12345345345345\dots$
- To convert recurring decimals to fractions we follow this procedure:

$$\begin{array}{ll}
 x = 0.23456456456\dots & \text{place equal to } x, \\
 100x = 23.456456456\dots & \text{leave only pure recurring section,} \\
 100000x = 23456.456456456\dots & \text{move decimal place to leave same recurring bit,} \\
 99900x = 23433 & \text{subtract to eliminate recurring bit,} \\
 x = \frac{23433}{99900} = \frac{7811}{33300} & \text{divide and simplify.}
 \end{array}$$

- Your fraction work and estimating should be fine. Read the appropriate section on pages 4–6. Remember when dealing with mixed fractions like  $4\frac{2}{3}$  it is usually easiest to convert to top-heavy fractions  $\frac{14}{3}$  before performing any calculations and then convert back at the end.

## Percentages & Ratios

- *All* percentage questions are most easily solved by making a table of percentage and amount and then cross multiplying and solving for the unknown. For example, I bought a car 3 years ago and it has since lost 41% of its value. It is now valued at £11800. What was its original value?

AMOUNT	PERCENTAGE
$x$	100
11800	59

So  $59x = 1180000$ , therefore  $x = \text{£}20000$ .

- Compound interest is most easily summed up by the formula  $A = P \times (1 + \frac{r}{100})^n$ , where  $P$  is the original amount invested,  $A$  is the total amount including new interest,  $r$  is the interest rate and  $n$  is the number of years invested. For example, I invested an amount of money 7 years ago at 4.5% compound interest and there is now £4082.59 in my account. How much did I invest? Put the numbers into the formula and solve;

$$4082.59 = P \times 1.045^7 \quad \text{so} \quad P = \frac{4082.59}{1.045^7} = \text{£}3000.$$

- Ratios are usually canceled down to their lowest form in integers. For example  $5 : 15 : 20$  goes to  $1 : 3 : 4$  and  $2 : \frac{1}{3}$  goes to  $6 : 1$ .
- In question where they say  $3 : 4 = x : 16$  it is usually pretty obvious to see what to do. In this case  $x = 12$  because the right hand is four times the left hand.

- When dividing quantities in a ratio you sum the components of the ratio and divide into the quantity. You then multiply the original ratio by this number. For example divide £154 in the ratio 3 : 4 : 4.

$$3 + 4 + 4 = 11 \quad \Rightarrow \quad \frac{154}{11} = 14 \quad \Rightarrow \quad (3 \times 14) : (4 \times 14) : (4 \times 14) = \text{£}42 : \text{£}56 : \text{£}56.$$

- The other way questions can be posed are when you are told that the ratio of men : women : children at a school play is 2 : 4 : 7 and that there are 84 children then we can easily see that 24 : 48 : 84, so there are 156 people in total at the play. (The key is spotting that  $7 \times 12 = 84$ . i.e. you need to find the number that multiplies 7 to make 84 and multiply the whole ratio by that number.)
- Two quantities are in direct proportion if when one doubles, the other doubles. When one triples, the other triples etc. For example, if told that  $x$  and  $y$  are in direct proportion and that  $x = 3$  when  $y = 5$ , what is  $x$  when  $y = 6$ ? We multiply 5 by  $\frac{6}{5}$  to make 6, so  $x = \frac{6}{5} \times 3 = 3.6$ .
- Two quantities are in inverse proportion if when one doubles the other halves. (One goes up, the other comes down!) So whatever I times one variable by from one state to the other I must *divide* the other.

## Equations

- With simple equations we must get the  $x$ 's to one side and the numbers to the other. We must do this by doing the *same thing to both sides*. For example:

$$\begin{aligned} 2x + 15 &= 5x - 3 \\ 15 &= 3x - 3 && \text{subtracting } 2x, \\ 18 &= 3x && \text{adding } 3, \\ x &= 6 && \text{dividing by } 3. \end{aligned}$$

- When equations have denominators (e.g.  $\frac{q+3}{5} - \frac{2q-3}{4} = \frac{q+1}{2} + 2$ ) we can get rid of them by multiplying by the entire equation by the LCM of the denominators. So in this example there are 4 terms in the equation so I must make sure I multiply *all* 4 of them by 20. The denominators will then *all* cancel with what you are multiplying by.

$$\begin{aligned} \frac{q+3}{5} - \frac{2q-3}{4} &= \frac{q+1}{2} + 2 \\ 20 \times \frac{(q+3)}{5} - 20 \times \frac{(2q-3)}{4} &= 20 \times \frac{(q+1)}{2} + 20 \times 2 \\ 4(q+3) - 5(2q-3) &= 10(q+1) + 40 \\ 4q + 12 - 10q + 15 &= 10q + 10 + 40 \\ -16q &= 23 \\ q &= -1\frac{7}{16}. \end{aligned}$$

- When equations have the unknown in the denominator we must multiply by the denominator. For example

$$\begin{aligned} \frac{20}{2x+1} &= 4 \\ (2x+1) \times \frac{20}{2x+1} &= 4 \times (2x+1) \\ 20 &= 8x + 4 \\ x &= 2. \end{aligned}$$

- With quadratic equations we use the same technique as with other simpler equations; i.e. get the  $x^2$  or the  $(\dots)^2$  to one side and the numbers to the other and then remember to take the positive *and* negative square roots. For example

$$\begin{aligned}4(x + 5)^2 + 5 &= 69 \\4(x + 5)^2 &= 64 \\(x + 5)^2 &= 16 \\x + 5 &= \pm 4 \\x &= -1 \text{ or } x = -9.\end{aligned}$$

If you end up being left with  $x^2 = \frac{49}{16}$  then take the square root of the top and the bottom to get  $x = \pm \frac{\sqrt{49}}{\sqrt{16}} = \pm \frac{7}{4} = \pm 1\frac{3}{4}$ .

- Trial and improvement is covered on a separate handout. The only important point to make is that if you want to find a solution to 2dp (say) then you sandwich the answer between (say)  $x = 3.45$  and  $x = 3.46$  then you *must* test  $x = 3.455$  and interpret the result to give either  $x = 3.45$  (to 2dp) or  $x = 3.46$  (to 2dp).

## Basic Algebra & Powers

- To factorise an expression you must ‘pull out’ the HCF of any coefficients and the *lowest* powers of any common variables. For example  $6x^3yz - 10x^2z = 2x^2z(3xy - 5)$ . Be careful of ones like  $6x^2 + 3x = 3x(2x + 1)$ ; notice the one at the end. You can always check your answers in any factorisation questions by just multiplying back out.
- You must be able to simplify expressions by multiplying out and collecting ‘like terms’. For example

$$3(2x - 4) - (3x - 4) - 3(3x + 2) = 6x - 12 - 3x + 4 - 9x - 6 = -6x - 17.$$

- Appreciate that  $x^4 = x \times x \times x \times x$  and  $x^{-3} = \frac{1}{x^3} = \frac{1}{x \times x \times x}$ . Also  $x^0$  is 1 for all  $x \neq 0$ . Three important index laws are

1.  $a^x \times a^y = a^{x+y}$ ,
2.  $\frac{a^x}{a^y} = a^{x-y}$ ,
3.  $(a^x)^y = a^{xy}$ .

They can be used to simplify expressions like

$$\frac{(2x)^2 \times 4x^2y^2}{8x^3y^3} = \frac{16x^4y^2}{8x^3y^3} = \frac{2x}{y} \quad \text{and} \quad \frac{4x^2y + 2x^3y^2}{2xy} = \frac{4x^2y}{2xy} + \frac{2x^3y^2}{2xy} = 2x + x^2y.$$

- Recognise the difference between equations, formulae, identities and expressions. An equation has only one variable and is correct for only one value of that variable. An identity is true for all values that the variable can take. A formula is an equation with more than one variable. An expression is a collection of terms with no ‘=’ sign. Below is a table of examples:

EQUATIONS	IDENTITIES	FORMULAE	EXPRESSIONS
$2x + 3 = 10$	$2(x + 3) = 2x + 6$	$E = mc^2$	$mx + c$
$2x + 4 = 3x - 2$	$x + y = y + x$	$F = ma$	$\frac{1}{2}mv^2$
$2(x + 4) = 3x - 10$	$(x + 2)^2 = x^2 + 4x + 4$	$y = mx + c$	$2a + 4b$

- You must be able to calculate values from a formula given values. For example, given  $E = \frac{1}{2}mv^2$ , calculate  $E$  when  $m = 2$  and  $v = -5$ . So  $E = \frac{1}{2} \times m \times v \times v = \frac{1}{2} \times 2 \times (-5) \times (-5) = 1 \times 25 = 25$ .

## Sequences & Formulae

- When given a sequence of numbers it is often useful to look at the difference between terms to find a pattern. For example given the triangular numbers we can see the difference has a readily identifiable pattern.

△ Numbers	1	3	6	10	15	21	28
Difference		+2	+3	+4	+5	+6	+7

We can use this pattern to work out the next few terms.

- When there is a *constant difference* between the terms then the  $n^{\text{th}}$  term ( $T_n$ ) is given by

$$T_n = (\text{diff. between terms})n + (\text{zeroth term}).$$

So for example

$$\begin{aligned}5, 8, 11, 14, 17 \dots &\implies T_n = 3n + 2, \\5, 1, -3, -7, -11 \dots &\implies T_n = -4n + 9, \\-3, -1, 1, 3, 5 \dots &\implies T_n = 2n - 5.\end{aligned}$$

- *Example 1*; In the sequence  $-4, 1, 6, 11, \dots$  what is the two hundredth term? Well the formula is clearly  $T_n = 5n - 9$  so the two hundredth term is  $T_{200} = 5 \times 200 - 9 = 991$ .
- *Example 2*; In the sequence  $T_n = 3n - 7$  one of the terms is 284. Which term is it? Just solve

$$284 = 3n - 7 \quad \Rightarrow \quad 291 = 3n \quad \Rightarrow \quad 97 = n.$$

So 284 is the 97<sup>th</sup> term. You must be good at questions like either of these examples.

- You need to be able to change the subject of an equation. This we do by isolating the desired letter. For example make  $x$  the subject in  $z = \frac{x+y}{x-y}$ :

$$z = \frac{x+y}{x-y} \quad \Rightarrow \quad z(x-y) = x+y \quad \Rightarrow \quad zx - x = y + zy \quad \Rightarrow \quad x = \frac{y+zy}{z-1}.$$

- Formulae can be manipulated to make them more useful. Given that  $y = ax + c$  and  $x = 2a + c$  we can write  $y$  in terms of  $a$  and  $c$  by substituting in:  $y = a(2a + c) + c = 2a^2 + ac + c$ .

## Linear & Quadratic Graphs

- To draw a straight line graph from an equation we can draw a table of  $x$  and  $y$ . For example draw the line  $y = -2x - 4$  we fill in the table

$x$	0	2	-3
$y$	-4	-8	2

We would then plot the points  $(0, -4)$ ,  $(2, -8)$  and  $(-3, 2)$  and join them up with a ruler to find the line.

- A faster way is to look at the line in the form  $y = mx + c$  where  $m$  is the gradient and  $c$  is the  $y$ -axis intercept. So the line  $y = 4x - 2$  has intercept  $-2$  and gradient 4. So it goes through the point  $(0, -2)$ . From this we go across *one to the right* and *up four* to find our next point (and so on).

- If the line is not given to you with  $y$  as the subject then you could make it the subject. For example

$$4x + 3y = 6 \quad \Rightarrow \quad y = \frac{6 - 4x}{3}.$$

Then do the  $xy$  table as above. However you could also just look at the equation  $4x + 3y = 6$  let  $x = 0$  to discover  $y = 2$  and then let  $y = 0$  to discover  $x = 1\frac{1}{2}$ . Therefore the line must go through  $(0, 2)$  and  $(1\frac{1}{2}, 0)$  so we can draw the line.

- Know how to read from conversion graphs and real world graphs (see pages 55 & 56). Know that on distance–time graphs you are often asked for speeds (=dist/time).
- To graph quadratic functions you need to draw a detailed table, devoting a column for each section of the equation. For example to draw  $y = 2x^2 - 2x - 4$  make the following table:

$x$	$2x^2$	$-2x$	$-4$	$y$
-3	18	6	-4	20
-2	8	4	-4	8
-1	2	2	-4	0
0	0	0	-4	-4
1	2	-2	-4	-4
2	8	-4	-4	0
3	18	-6	-4	8

And then plot the points  $(-3, 20)$ ,  $(-2, 8)$ ,  $\dots$ ,  $(3, 8)$  and join with a smooth curve. You should find that it is a U-shaped curve crossing the  $x$ -axis at  $-1$  and  $2$ .

- If the number of  $x^2$ 's is positive (e.g.  $y = 2x^2 + \dots$ ) then the graph will be U-shaped and if the number of  $x^2$ 's is negative (e.g.  $y = -3x^2 + \dots$ ) then the graph will be  $\cap$ -shaped.

## Pythagoras & Circle Properties

- Pythagoras' Theorems states that in a *right angled* triangle with sides  $a$ ,  $b$  and  $c$  where  $c$  is the hypotenuse (longest length; always opposite the right angle) then  $a^2 + b^2 = c^2$ . In other words "the short one squared plus the short one squared equals the long one squared".
- You will always know two of the sides and not know the other. Assign a letter to the unknown side and then write down an equation for the triangle. Solve for the unknown and *don't forget to square root at the end!* For example you might get

$$x^2 + 5^2 = 9^2 \quad \Rightarrow \quad x^2 = 81 - 25 \quad \Rightarrow \quad x = \sqrt{56}.$$

- Pythagoras can be used to work out the length between two points in the  $xy$ -plane. For example calculate the length between  $(-3, 5)$  and  $(2, 6)$  is  $\sqrt{5^2 + 1^2} = \sqrt{26}$ .
- Learn the properties of circles on page 60.
- The area of a circle is given by  $A = \pi r^2$ . Make sure you are using the radius in this formula and *not* the diameter.
- The circumference of a circle is given by  $C = 2\pi r$  (or, if you like,  $C = \pi d$ ).
- The tangent to a circle and the radius at that point are *always* at right angles.
- Tangents drawn from a point to both sides of a circle are equal in length. This implies that there will exist a kite with two right angles in it. Cut it in two to form a right angled triangle.

## Trigonometry

- In trigonometry you will always be referring to *right angled* triangles. (If there isn't one then you will need to construct one somehow.) There will always be two sides and one angle that you are 'interested in'. You will know two and you will need to work out the third. Assign the unknown length or angle a letter (usually  $a$  for angle and  $x$  for length).
- All you need to remember is SOHCAHTOA, which is a way of remembering that

$$\sin a = \frac{\text{opposite}}{\text{hypotenuse}} \quad \cos a = \frac{\text{adjacent}}{\text{hypotenuse}} \quad \tan a = \frac{\text{opposite}}{\text{adjacent}}.$$

The hypotenuse is fixed, and the adjacent and opposite lengths are relative to the angle you are 'interested in'.

- Then write down the equation and solve for the unknown (it will always be sin, cos or tan of an angle equals a length over a length). Here are three initial equations and their solutions; make sure you can do all of them.

$$\begin{array}{lll} \sin a = \frac{4}{5} & \cos 40 = \frac{x}{3} & \tan 70 = \frac{15}{x} \\ a = \sin^{-1}\left(\frac{4}{5}\right) & x = 3 \cos 40 & x = \frac{15}{\tan 70} \\ a = 53.1 \text{ (3sf)} & x = 2.30 \text{ (3sf)} & x = 5.46 \text{ (3sf)} \end{array}$$

- The rule of thumb is that when the angle is unknown you will need to use the inverse function. For example if  $\tan a = \frac{3}{7}$  then  $a = \tan^{-1}\left(\frac{3}{7}\right)$
- Bearings are always given with *three* digits (e.g.  $004^\circ$ ) and are measured clockwise from north. Make sure you read the question very carefully and draw in as many north lines as possible. These are all parallel so you can use your parallel line angle facts to help solve problems.

## Transformations

- In any **transformation** in this section the 'object' is the original shape and the 'image' is the object after transformation.
- A **translation** is a sliding movement in the  $xy$ -plane without rotation, reflection or enlargement. It is usually described by a column vector of the form  $\begin{pmatrix} \text{movement right} \\ \text{movement up} \end{pmatrix}$ . So the vector  $\begin{pmatrix} 4 \\ -3 \end{pmatrix}$  is a translation 4 right and 3 *down*.
- To translate an object by a given column vector (e.g.  $\begin{pmatrix} -3 \\ 5 \end{pmatrix}$ ) it is best to consider *one* point on the shape and move it by the vector (3 left and 5 up in this case) and then redraw the rest of the shape from that point.
- If you are given two shapes and it has been translated, then to describe the translation we consider the same point on both shapes and see how it has moved. We then say that the transformation is a translation and state the column vector.
- **Rotations**. Three things define a rotation;
  - The *centre of rotation*. That is the point about which your shape is rotated.
  - The *direction* of rotation. Either clockwise  $\curvearrowright$  or anti-clockwise  $\curvearrowleft$ .
  - The *size* of the angle through which the shape is rotated. For example  $45^\circ$ ,  $90^\circ$ ,  $180^\circ$ .

- **Reflections.** To reflect a shape in a line we must make sure that for *each point* on the object we go *to* the line at right angles and then go the *same* distance again to find the equivalent point on the image.
- An **enlargement** changes the size (but not the shape) of an object. The image of an enlargement is said to be *similar* to the object. The *scale factor* describes how many times larger the shape has become. For example if the scale factor is three then *all* the lengths on the object are three times longer in the image.
- There is also the centre of enlargement to consider. If you are given two similar shapes and asked to find the centre of the enlargement then you just ‘ray trace’ from common points on object and image. Where these lines cross is the centre of enlargement.
- You must also know how to enlarge a shape with a given scale factor and centre. You take a single point on the image and work out the column vector *from* the centre *to* the point (e.g.  $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ ). You then multiply the vector by the scale factor (e.g. for a scale factor of two;  $2 \times \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$ ). The image point will then be  $\begin{pmatrix} 4 \\ 6 \end{pmatrix}$  from the centre. Given that the scale factor is two, you then redraw the whole shape from that point but with all lengths twice as long.
- When asked to describe an enlargement you must include both the centre of enlargement and the scale factor (and also, of course, that it is an enlargement!). If there is a negative scale factor then the shape will also have been rotated and not have the same orientation, as it does with positive scale factors.
- In a **scale drawing** the *scale* is a ratio showing how real life relates to the scale drawing. For example 1 : 25000 tells us 1cm in the map is 25000cm on the ground. Therefore 1cm = 250m and 4cm = 1km.

## Mensuration

- You need to know the formulae for the areas of certain simple plane shapes. You may also be presented by ‘compound’ shapes that are constructed by pasting together common shapes. They are summarised thus:

TRIANGLE	$A = \frac{1}{2}bh$	$h$ must be perpendicular height
PARALLELOGRAM	$A = bh$	$h$ must be perpendicular height
TRAPEZIUM	$\left(\frac{a+b}{2}\right)h$	$h$ must be perpendicular height & $a$ and $b$ must be the parallel sides
CIRCLE	$A = \pi r^2$	Don't confuse radius and diameter

- Volume and Surface Area<sup>1</sup> of *cuboids* are obvious;

$$\text{Volume} = lbh \quad \text{S.A.} = 2(lb + lh + hb).$$

- The volume of a prism is the (area of the *extended* face)  $\times$  (its length). It is almost *always* best to calculate the area of the face first (no matter how fiddly) and *then* multiply by the length. The surface area is then done on an *ad hoc*<sup>2</sup> basis.
- Be on the lookout for shapes where bits are taken away from some larger shape. For example *Thin Book, Ex 10G, the ‘tent’ question*. The face is a 10 by 8 rectangle with two quarter circles removed. Therefore the area of the face is  $A = (10 \times 8) - \frac{1}{2}(\pi \times 4^2)$ .

<sup>1</sup>Surface Area will henceforth be referred to as S.A.

<sup>2</sup>*ad hoc* = Fly by the seat of your pants.



- Cylinders are prisms with circular cross section.  $\text{Vol} = \pi r^2 h$  and  $\text{S.A.} = 2\pi r^2 + 2\pi r h$ . In S.A. the  $2\pi r^2$  is the area of the circular top and bottom; the  $2\pi r h$  is the area of the ‘curved face’.<sup>3</sup>
- When faced with a calculation in mensuration you *must* be careful about units. To calculate an area or volume you will need to convert all lengths to one consistent unit (e.g. mm, cm, m, km, etc.).
- You also need to be able to convert between units. The basic conversions you must know are:

1 kg	2.2 lb (pounds)	2.54 cm	1 inch
8 km	5 miles	4.5 litres	1 gallon
1 litre	1.75 pints	1 litre	1000 cm <sup>3</sup>
30 cm	1 ft		

- You must also know more complicated conversions; this is best illustrated by way of example. Convert  $5.4 \text{ kg/cm}^3$  to  $\text{lb/ft}^3$  (this is a *very dense material!*). We write

$$5.4 \text{ kg/cm}^3 = \frac{5.4 \times (1 \text{ kg})}{(1 \text{ cm})^3} = \frac{5.4 \times (2.2 \text{ lb})}{\left(\frac{1}{30} \text{ ft}\right)^3} = \frac{5.4 \times 2.2}{\left(\frac{1}{30}\right)^3} \text{ lb/ft}^3 = 320,760 \text{ lb/ft}^3.$$

## Handling Data I

- Scatter graphs can be used to see how two sets of data are related. If the points are almost in a line then there is a high degree of *correlation* between the variables. We would expect high correlation between people’s times for the 100m sprint and 200m sprint (fat people are slow at both etc.)
- There is *positive correlation* if, as one variable increases, so does the other. There is *negative correlation* if, as one increases, the other decreases. You can usually draw the *line of best fit*. You can use this to estimate values from the graph. The line of best fit does **not** always go through the origin (0,0).
- Given a frequency distribution you create a new column (the product of the first two;  $xf$ ) and sum it to help you work out the *mean*. For example

$x$	$f$	$\Rightarrow$	Create	$\Rightarrow$	$x$	$f$	$xf$
0	1	$\Rightarrow$	Extra	$\Rightarrow$	0	1	0
1	3	$\Rightarrow$	Column	$\Rightarrow$	1	3	3
2	5	$\Rightarrow$	&	$\Rightarrow$	2	5	10
3	4	$\Rightarrow$	Sum	$\Rightarrow$	3	4	12
4	2	$\Rightarrow$		$\Rightarrow$	4	2	8
	15					15	33

So the mean is  $\bar{x} = \frac{\sum xf}{\sum f} = 33/15$ .

- If you have grouped data ( $0 \leq x < 5$ ,  $5 \leq x < 10$ , etc.) then the best you can do is *estimate the mean* by using the mid-point (2.5, 7.5, etc.) and carry out the same process as above.
- The *modal class* will be the class with the highest frequency.

<sup>3</sup>Curved Face = The bit where the sticker goes on a can of beans.

- Find the *median* of a set of data. The median occurs at the  $(\frac{n+1}{2})$ th data point. For example, median of 2, 3, 4 is 3, but the median of 1, 2, 3, 4 is 2.5 (the average of 2 & 3). For grouped data you can only state the class interval that contains the median (e.g. it lies in the class  $15 \leq x < 20$ ).
- Read pages 101–3 about time series and moving averages. The moving average will always be plotted on a new graph.

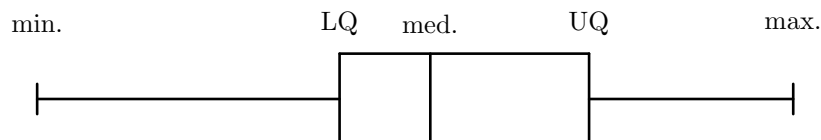
## Handling Data II

- The *lower quartile* is the piece of data one quarter of the way in to a set of data. The *upper quartile* is the piece of data three quarters of the way in. The *interquartile range* is defined as the upper quartile – lower quartile. It is a measure of **spread** (how spread out the data is).
- The cumulative frequency is a “running total” of the frequencies as you go up the values. For example

$x$	$f$		$x$ (upper limit of)	cum. freq
$0 \leq x < 5$	8	⇒ Create	5	8
$5 \leq x < 10$	13	⇒ Cumulative	10	21
$10 \leq x < 15$	17	⇒ Frequency	15	38
$15 \leq x < 20$	10		20	48

Plot the second of these tables and join it with a smooth curve to form the *cumulative frequency curve*. From this the median and the two quartiles can be found. Page 107.

- Once these values are found we can draw a *box and whisker diagram*. The box and whisker diagram uses five values: the minimum, the maximum, the lower quartile, the upper quartile and the median. It is good for showing spread and comparing two quantities.



- Histograms are usually drawn for continuous data in classes. If the classes have equal widths, then you merely plot amount against frequency.
- If the classes do *not* have equal widths then we need to create a new column for *frequency density*. Frequency density is defined by  $f.d. = \frac{\text{frequency}}{\text{class width}}$ . However the f.d. may be scaled to make the numbers ‘nicer’. The relationship therefore becomes

$$f.d. = \alpha \times \frac{\text{frequency}}{\text{class width}}$$

where  $\alpha$  is a scale factor that may need to be determined. Read 113–5.

- The *area* of the bars are what represents the frequency, *not* the height.
- Know the difference between *primary* and *secondary* data. Primary data is collected through experiments, surveys, questionnaires etc. Secondary data is data extracted from published sources.
- If you have a piece of data that says a child weighs 1250 kg it’s probably best to discard it. (Use common sense with unusual data.)