Specimen of Written Test at Interview

Issued May 1999

(Please note that the format of this test may change)

THE COLLEGES OF OXFORD UNIVERSITY

MATHEMATICS

Time Allowed: $2\frac{1}{2}$ hours

For candidates applying for Mathematics, Mathematics and Philosophy, Mathematics and Computation, and Computation.

Name: **College of preference:**

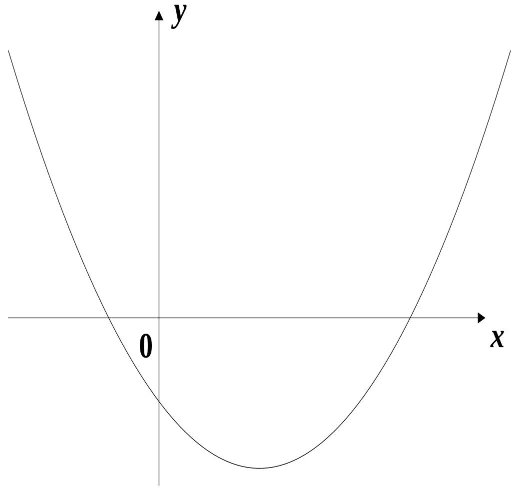
Answer all questions.

Place your answers to Question 1 in the table below. Write your answers to Questions 2 to 5 in the space provided. Additional sheets of paper may be inserted.

1. Place a tick (✓) in the appropriate box.

Question 1 Part	Answer			
	(i)	(ii)	(iii)	(iv)
(a)				
(b)				
(c)				
(d)				
(e)				
(f)				
(g)				
(h)				
(j)				
(k)				

For each part choose the correct answer from (i)–(iv). There is only one in each case.



(a) The diagram above shows the graph of the function $y = ax^2 + bx + c$. Then:

(i) $b^2 - 4ac > 0$; (ii) $b^2 - 4ac = 0$; (iii) $b^2 - 4ac \leq 0$; (iv) $b^2 - 4ac < 0$.

(b) The inequality $2^n > n^2$ is true for:

(i) no integers $n \geq 0$; (ii) all integers $n \geq 0$; (iii) all integers $n > 4$; (iv) all integers $n \geq 4$.

(c) The simultaneous equations

$$\begin{aligned} ax + by &= 1 \\ cx + dy &= 0 \end{aligned}$$

in x and y:

(i) have a solution whatever the values of a, b, c, d may be;

(ii) have a unique solution whatever the values of a, b, c, d may be;

(iii) have a solution only if $ad \neq bc$;

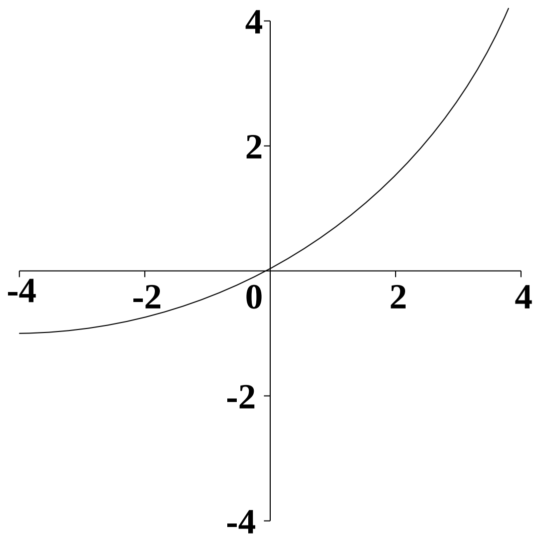
(iv) have a unique solution only if $ad \neq bd$.

(d) The complete set of solutions of the equation $\sin 2x = \cos x$ in the range $0 \leq x \leq 2\pi$ is:

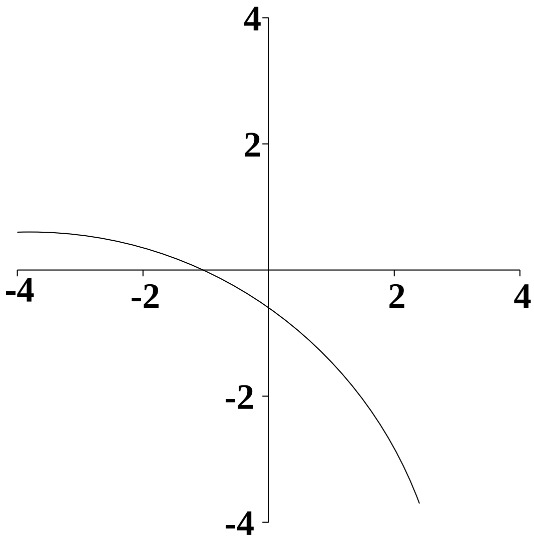
(i) $\{\pi/2,\, 3\pi/2\}$; (ii) $\{\pi/6,\, 5\pi/6\}$; (iii) $\{\pi/6,\, \pi/2\}$; (iv) $\{\pi/6,\, \pi/2,\, 5\pi/6,\, 3\pi/2\}$.

(e) If $|x - 3| < 1$ and $|x - 1| > 2$, then:

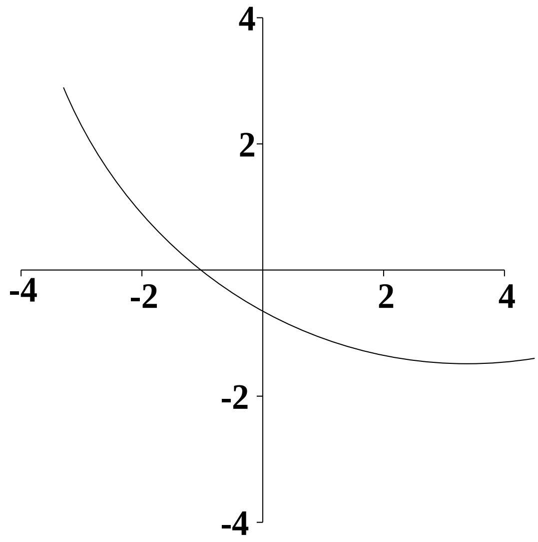
(i) $-1 < x < 4$; (ii) $3 < x < 4$; (iii) $2 < x < 3$; (iv) $2 < x < 4$.



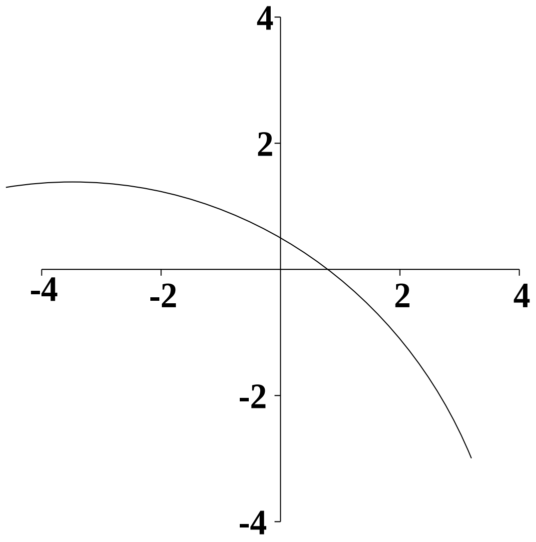
(f) The diagram above shows the graph of the function $y = f(x)$. The graph of the function $y = -f(x+1)$ is:



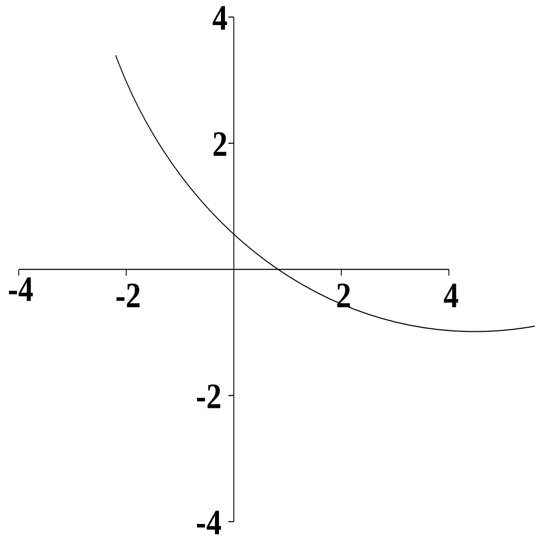
(i)



(ii)



(iii)



(iv)

(g) As n becomes very large and positive, $10000^{-\frac{1}{n}}$ approaches:

(i) 0; (ii) 1; (iii) 10000; (iv) ∞.

(h) The derivative of the function $y = (e^{\cos(5x)})^2$ is:

(i) $-5\sin(5x)(e^{\cos(5x)})^2$; (ii) $-20\sin(5x)\cos(5x)(e^{\cos(5x)})^2$;

(iii) $-10\sin(5x)(e^{\cos(5x)})^2$; (iv) $-10\sin(5x)\cos(5x)(e^{\cos(5x)})^2$.

(j) The derivative of the function

$$F(x) = \int_0^x f(t) \quad dt$$

is:

(i) $f(x) - f(0)$; (ii) $f'(x)$; (iii) $f(x)$; (iv) $f'(x) - f'(0)$.

(k) An entrance candidate is dealt three cards from a pack of fifty-two playing cards. To one significant figure the probability that he receives exactly one king is:

(i) 0.003; (ii) 0.01; (iii) 0.2; (iv) 0.05.

[There are four kings in a pack of playing cards.]

2. (a) Factorise the expresssion $x^2 + x - 6$.

(b) For which values of the real constant a does the equation

$$x^2 + x - a = 0$$

have at least one real solution? Write down these solutions in terms of a.

(c) Show that, for any value of the real constant b, the equation

$$x^3 - (b + 1)x + b = 0$$

has $x = 1$ as a solution. Find all values of b for which this equation has exactly two distinct solutions.

3. (a) Write down the equation of the straight line through the point (1,2) with slope -1.

(b) Let l be a line with equation

$$y = (2 - a) + ax,$$

where a is a constant. Show that, for any a, the line passes through the point (1,2). Find the equation of the line perpendicular to this line which also passes through the point (1,2).

(c) Find the equations of the lines which pass through the point (1,2) and have perpendicular distance 1 from the origin.

4. (a) Find the values of

1. $\int_{-1}^{1} (x^2 - x)\, dx$,
2. $\int_{-1}^{1} (x^3 + x^2 - 2x)\, dx$.

(b) Sketch the graph of $y = x^2 - x$ and indicate which difference in areas is reporesented by your answer to (a)(i).

(c) Find the total area (measured positively) that lies between the graphs of $y = x^2 - x$ and $y = x^3 + x^2 - 2x$ between $x = -1$ and $x = 1$.

(d) The answers to (a)(i) and (a)(ii) ar related in a particular way. Explain how the relationship can be seen *without* working out any integrals.

5. A total of 12 noughts and 4 crosses are arranged in 4 rows of 4. One such arrangement is illustrated below.

0	0	×	0
0	×	0	×
0	0	0	0
×	0	0	0

(a) How many arrangements are there altogether?

(b) How many arrangements are there in which there is a cross in every row?

(c) How many arrangements are there in which there is a cross in every row and in every column?