

# EDEXCEL FOUNDATION

Stewart house 32 Russell Square London WC1B 5DN

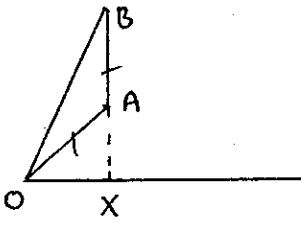
June 2003

Advanced Extension Award

General Certificate of Education

Subject: AEA

Paper: 9801

Question Number	Scheme	Marks
1.	 <p> <math>\Delta OAB</math> is isos (since <math> OA  =  AB  = 1</math>)  <math>\angle OAB = \frac{3\pi}{4}</math> (since <math> OX  =  AX </math> and <math>\therefore \angle AOX = \frac{\pi}{4}</math>)  <math>\therefore \angle BOA = \frac{\pi}{8}</math> or <math>\angle BOX = \frac{3\pi}{8}</math> </p> $\tan \frac{3\pi}{8} = \frac{BX}{OX} = \frac{1 + \frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = \underline{\underline{\sqrt{2} + 1}} \quad (*)$	<p>B1 B1 B1 M1, A1 cso (5)</p>
2. (See ALT)	$2 \sin^2 \theta - \tan \theta = 2 \sin 2\theta - 2$ $2 \tan^2 \theta - \tan \theta \sec^2 \theta = \frac{2 \sin 2\theta}{\cos^2 \theta} - 2 \sec^2 \theta$ $2 \tan^2 \theta - \tan \theta (1 + \tan^2 \theta) = 4 \tan \theta - 2(1 + \tan^2 \theta)$ $t = \tan \theta \Rightarrow 0 = t^3 - 4t^2 + 5t - 2$ $0 = (t-1)(t^2 - 3t + 2)$ $0 = (t-1)(t-1)(t-2)$ $\therefore \underline{\underline{\tan \theta = 1 \text{ or } \tan \theta = 2}}$	<p>M1 M1 M1 M1 Correct tidied cubic in tan <math>\theta</math> A1 1st factor M1 All factors A1 A1, A1 (8)</p>
	<p><u>Style, Clarity &amp; Presentation</u></p> <p>(a) <u>5 marks</u>            For a novel or neat solution to any of questions 2-7. Apply once per question in upto 3 questions.            S2 if solution is fully correct in principle, elegance and accuracy            S1 if principle is sound but a minor algebraic or numerical slip occurs.</p> <p>(b) <u>1 mark</u>            For a good and largely accurate attempt at the whole paper.</p>	<p>S6 (S2 x 3) T1 (7)</p>

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3.	$\frac{dy}{dx} = \frac{2t}{3t^2} = \frac{2}{3t}$ <p>c.p. <math>t=2</math>, <math>\therefore</math> gradient of tangent <math>= \frac{1}{3}</math></p> <p>Equation of tangent is: <math>y - 4 = \frac{1}{3}(x - 8)</math> or <math>3y = x + 4</math></p> <p>Meets C &amp; Q i.e. <math>3t^2 = t^3 + 4</math> <span style="float: right;"><i>curve = eqn</i></span></p> $0 = (t-2)(t^2 - t - 2)$ $0 = (t-2)(t-2)(t+1)$ <p><math>\therefore</math> c.q. <math>t = -1</math> <span style="float: right;"><i>Full method to solve</i></span></p> $\int y dx = \int t^2 \frac{dx}{dt} dt = \int t^2 \cdot 3t^2 dt = \int 3t^4 dt$ <span style="float: right;"><i>suitable <math>\int</math></i></span> $\text{Area} = \int_{t=-1}^{t=2} 3t^4 dt = \left[ \frac{3t^5}{5} \right]_{-1}^2 = \left( \frac{96}{5} \right) - \left( -\frac{3}{5} \right)$ <span style="float: right;"><i>Correct function</i></span> $= 19.8$ <p>Required area is Trapezium - 19.8.</p> <p>Area of trapezium <math>= \frac{1}{2} (1+4) \times 9 = 22.5</math></p> <p><math>\therefore</math> required area <math>= 22.5 - 19.8 = \underline{\underline{2.7}}</math></p>	<p>M1</p> <p>A1</p> <p>M1 (3)</p> <p>M1</p> <p>M1</p> <p>A1 (3)</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>B1</p> <p>A1 (5)</p> <p style="text-align: right;">(11)</p>

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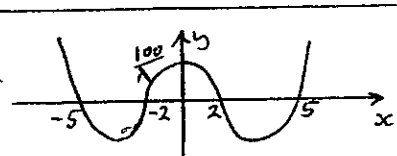
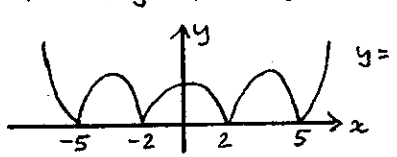
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4. (a)	$1-3x \equiv (Ax+B)(1-x)^2 + C(1+3x^2)(1-x) + D(1+3x^2)$ $x=1 \Rightarrow -2 \equiv 4D \quad \therefore D = -\frac{1}{2}$ $x=0 \Rightarrow 1 = B+C+D \quad \text{or} \quad B+C = \frac{3}{2} \quad \textcircled{1}$ $\text{Coeff } x^3: 0 = A-3C \quad \text{or} \quad A = 3C \quad \textcircled{2}$ $\text{Coeff } x: -3 = A-2B-C \quad \textcircled{3}$ $\textcircled{2} \text{ into } \textcircled{3} \Rightarrow -\frac{3}{2} = C-B, \text{ solving with } \textcircled{1} \Rightarrow \underline{C=0, A=0, B=\frac{3}{2}}$	M1 B1 M1 Sufficient suitable equations A2/1/0 (5)
(b)	$f(x) \equiv \frac{3}{2}(1+3x^2)^{-1} - \frac{1}{2}(1-x)^{-2}$ $= \frac{3}{2} \left[ 1-3x^2 + \frac{(-1)(-2)}{2!} 9x^4 \dots \right] - \frac{1}{2} \left[ 1+2x + \frac{(-2)(-3)}{2!} x^2 + \dots x^3 + \dots x^4 \right]$ $= \left( \frac{3}{2} - \frac{9}{2}x^2 + \frac{27}{2}x^4 \dots \right) - \left( \frac{1}{2} + x + \frac{3}{2}x^2 + 2x^3 + \frac{5}{2}x^4 \dots \right)$ $= \underline{1 - x - 6x^2 - 2x^3 + 11x^4 \dots}$	M1 Use of Bin M1 Collecting Terms A2/1/0 (1000) (4)
(c)	$f'(x) = -1 - 12x \dots \quad \therefore f(0) = 1, f'(0) = -1$ $\therefore \text{equation of tangent is } \underline{y = 1 - x}$	f(0), f'(0) M1 A1 (2) <b>(11)</b>
5 (a)	 <p style="text-align: right;">Shape &amp; symmetry <math>\pm 5, \pm 2</math> <math>\frac{100}{\lambda}</math></p>	B1 B1 B1 (3)
(b)	$f'(x) = \frac{1}{\lambda} [(x^2-4) \cdot 2x + (x^2-25) \cdot 2x]$ $f'(x) = 0 \Rightarrow 2x(2x^2-29) = 0, \quad \therefore x = 0 \text{ (or)} \pm \sqrt{\frac{29}{2}}$ $\text{From diagram range is } f \geq f\left(\sqrt{\frac{29}{2}}\right), \quad \text{i.e. } \underline{f \geq -\frac{110 \cdot 25}{\lambda} \text{ or } -\frac{441}{4\lambda}}$	f' M1 f'=0 M1, A1 M1, A1 (5)
(c)	 <p style="text-align: right;"><math>y =  f(x) </math></p> $\underline{k=2} \text{ if } 2 > \frac{110 \cdot 25}{\lambda}, \text{ i.e. } \underline{\lambda > 56}$ $\underline{k=4} \text{ if } 4 = \frac{110 \cdot 25}{\lambda}, \lambda \notin \mathbb{Z}^+ \text{ No Sol}^n$	M1, A1 B1
	$\underline{k=6} \text{ if } \frac{100}{\lambda} < 6 < \frac{110 \cdot 25}{\lambda}; \text{ i.e. } \frac{100}{6} < \lambda < \frac{110 \cdot 25}{6} \text{ i.e. } \underline{\lambda = 17, 18}$ $\underline{k=7} \text{ if } \frac{100}{\lambda} = 7, \lambda \notin \mathbb{Z}^+ \text{ No Solution}$ $\underline{k=8} \text{ if } 8 < \frac{100}{\lambda}, \text{ i.e. } \lambda < \frac{100}{8} \text{ i.e. } \underline{0 < \lambda \leq 12}$	M1, A1, A1 B1 M1, A1 (9) <b>(17)</b>

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6 (a)	$\begin{aligned} (\text{LHS})^2 &= (2+\sqrt{3}) - 2\sqrt{(2+\sqrt{3})(2-\sqrt{3})} + (2-\sqrt{3}) \\ &= 4 - 2\sqrt{4-3} \\ &= 2 = (\text{RHS})^2 \end{aligned}$ <p>[<math>\because 2+\sqrt{3} &gt; 2-\sqrt{3}</math> we know LHS <math>&gt; 0</math>]</p>	<p>Attempt (LHS)<sup>2</sup></p> <p>M1 A1 A1 c.s.o. (3)</p>
(b)	$\log_{\frac{1}{8}} \sqrt{2} = \frac{\log_2 \sqrt{2}}{\log_2 \frac{1}{8}}, = \frac{\log_2 2^{\frac{1}{2}}}{\log_2 2^{-3}} = \frac{\frac{1}{2}}{-3} = -\frac{1}{6} \text{ (*)}$	<p>Change of base <math>2^{\frac{1}{2}}, 2^{-3}</math></p> <p>M1, M1, A1 c.s.o. (3)</p>
ALT (b)	$\log_{\frac{1}{8}} \sqrt{2} = x \Rightarrow \sqrt{2} = \left(\frac{1}{8}\right)^x$ <p>i.e. <math>2^{\frac{1}{2}} = 8^{-x} = (2^3)^{-x} = 2^{-3x}</math></p> <p><math>\therefore \frac{1}{2} = -3x</math> i.e. <math>x = -\frac{1}{6}</math></p>	<p>M1</p> <p>M1 cf powers of 2</p> <p>A1 c.s.o.</p>
(c) See ALT.	$\sqrt{a+\sqrt{15}} - \sqrt{a-\sqrt{15}} = \left(\frac{1}{n}\right)^{-\frac{1}{2}} = \sqrt{n}$ <p>Squaring <math>\Rightarrow a + \sqrt{15} - 2\sqrt{a^2-15} + a - \sqrt{15} = n</math></p> <p>i.e. <math>2(a - \sqrt{a^2-15}) = n</math></p> <p>So we require <math>a^2 - 15 = b^2</math></p> <p>i.e. <math>a^2 - b^2 = 15</math></p> <p>or <math>(a-b)(a+b) = 15</math></p> <p>But <math>15 = 3 \times 5</math> or <math>1 \times 15</math></p> <p><math>3 \times 5 \Rightarrow \begin{cases} a-b=3 \\ a+b=5 \end{cases} \therefore \underline{a=4}</math></p> <p><math>1 \times 15 \Rightarrow \begin{cases} a-b=1 \\ a+b=15 \end{cases} \therefore \underline{a=8}</math></p> <p><math>a=4, n = 2(4 - \sqrt{4^2-15}) = 6</math></p> <p><math>a=8, n = 2(8 - \sqrt{64-15}) = 2</math></p>	<p>Out of logs <math>\sqrt{n}</math></p> <p>M1 A1 M1 A1 M1 M1 Use of 'dots' M1 M1 Solving M1 A1 A1 A1 A1 A1 A1 A1 A1</p> <p><math>\therefore</math> Pairs are <math>\underline{a=4, n=6}</math> <math>\underline{a=8, n=2}</math></p> <p>(13)</p> <p style="text-align: center; border: 1px solid black; border-radius: 50%; width: 30px; margin: auto;">19</p>

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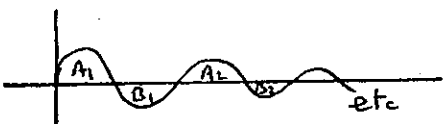
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7 (a)	$e^{-x} \sin x = 0 \Rightarrow \sin x = 0, x = 0, \pi$ <span style="float: right;"><math>\gg  x  &lt; \pi \Rightarrow \sin x = 0</math></span> $x$ coordinates of P, Q, R are <u><math>P = \pi, Q = 2\pi, R = 3\pi</math></u>	M1 A1 (2)
(b)	$I = [-e^{-x} \cos x] + \int e^{-x} (-\cos x) dx$ <span style="float: right;">Attempt Part</span> $= [-e^{-x} \cos x] - \{ [e^{-x} \sin x] - \int e^{-x} \sin x dx \}$ <span style="float: right;">Second Part Attempt</span> $I = -e^{-x} (\cos x + \sin x) - I$ <span style="float: right;">Recognize I</span> <u>i.e. <math>I = -\frac{1}{2} e^{-x} (\cos x + \sin x) + \text{constant}</math> *</u>	M1 A1 M1 M1 A1 c/w (5)
(c)	$A_1 = \int_0^\pi, A_2 = \int_{2\pi}^{3\pi} \dots \dots \dots A_n = \int_{(2n-2)\pi}^{(2n-1)\pi}$ <span style="float: right;">ODD <math>\pi</math></span> $A_n = \left[ \frac{1}{2} e^{-x} (\cos x + \sin x) \right]_{(2n-2)\pi}^{(2n-1)\pi}$ <span style="float: right;">EVEN <math>\pi</math></span> $= -\frac{1}{2} (-1+0) e^{-(2n-1)\pi} - -\frac{1}{2} (1+0) e^{-(2n-2)\pi} = \frac{1}{2} e^\pi e^{-2n\pi} + \frac{1}{2} e^{2\pi} e^{-2n\pi}$ <span style="float: right;">Apply limits</span>	M1 A1 M1 A1 (6)
(d)	$A_1 = \frac{1}{2} (e^\pi + e^{2\pi}) e^{-2\pi}, A_2 = \frac{1}{2} (e^\pi + e^{2\pi}) e^{-4\pi} \dots$ <span style="float: right;">Identify GP</span> $a = \frac{1}{2} (1 + e^\pi) \quad r = e^{-2\pi}$ <span style="float: right;">suitable a</span> $S_\infty = \frac{a}{1-r} = \frac{1}{2} \frac{(1 + e^{-\pi})}{1 - e^{-2\pi}} = \frac{1}{2} \frac{(1 + e^{-\pi})}{(1 - e^{-\pi})(1 + e^{-\pi})}$ <span style="float: right;">Use of <math>S_\infty</math></span> <u>i.e. <math>S_\infty = \frac{1}{2(1 - e^{-\pi})} = \frac{e^\pi}{2(e^\pi - 1)}</math></u>	M1 A1 A1 M1 A1 c/w (5)
(e)	 $\text{Let } S_A = \sum_1^\infty A_n; S_B = \sum_1^\infty B_n$ $\text{Given integral } \Rightarrow \frac{1}{2} = S_A - S_B$ $\text{We require } S \text{ where } S = S_A + S_B$ $\text{Adding } \Rightarrow S + \frac{1}{2} = 2S_A \quad \text{i.e. } S = 2S_A - \frac{1}{2}$ <u>i.e. <math>S = \frac{2e^\pi}{2(e^\pi - 1)} - \frac{1(e^\pi - 1)}{2(e^\pi - 1)} = \frac{e^\pi + 1}{2(e^\pi - 1)}</math> (p.e.)</u>	M1 (Use of $\int = \frac{1}{2}$ ) M1 M1 (System of $S_A$ ) A1 (4)

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## ALTERNATIVE SOLUTIONS

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2.	$\frac{\sin \theta}{\cos \theta} (2 \sin \theta \cos \theta - 1) = 2 (\sin 2\theta - 1)$ $\tan \theta (\sin 2\theta - 1) = 2 (\sin 2\theta - 1)$ $(\tan \theta - 2) (\sin 2\theta - 1) = 0$ $\therefore \sin 2\theta = 1 \Rightarrow \theta = 45, 225$ $\therefore \underline{\underline{\tan \theta = 1}}, \text{ or } \underline{\underline{\tan \theta = 2}}$	<p>sec <math>\theta = \frac{1}{\cos \theta}</math> + attempt factors M1</p> <p>(sin 2<math>\theta</math> - 1) factor M1</p> <p>Two factors M1 both correct in tan <math>\theta</math> or sin 2<math>\theta</math> A1</p> <p>M1, A1</p> <p>A1, A1</p> <p style="text-align: right;">(8)</p>
6(c) <u>TRIAL</u>	<p>175 marks as per scheme <math>\rightarrow</math> seek <math>a^2 - 15 = b^2</math></p> <p>Trial: <math>a=4 \Rightarrow a^2 - 15 = 1</math>  <math>a=5 \Rightarrow a^2 - 15 = 10</math>  <math>a=6 \Rightarrow a^2 - 15 = 21</math>  <math>a=7 \Rightarrow \quad \quad = 34</math>  <math>a=8 \Rightarrow \quad \quad = 49</math></p> <p>If <math>a=8 \Rightarrow n=2</math>  <math>a&gt;8 \Rightarrow n&lt;2</math></p> <p><math>n \neq 1</math> since if <math>n=1</math> <math>a - \sqrt{a^2 - 15} = 0.5</math>  and <math>(a - 0.5)^2 = a^2 - 15 \Rightarrow a = 15.25 \notin \mathbb{Z}</math></p> <p>Argument for <math>n &lt; 2, n &gt; 8</math> B1</p> <p>Argument for <math>n = 1</math> B1</p> <p>Possible pairs as in scheme (3)</p>	<p>(5)</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>B1</p> <p>B1</p> <p>(3)</p>
<u>SQUARING</u>	<p>174 marks as per scheme <math>\rightarrow 2a - \sqrt{a^2 - 15} = n</math> *</p> <p>Squaring <math>\rightarrow a = \frac{n^2 + 60}{4n}</math></p> <p><math>n=2 \Rightarrow (a=8)</math>  <math>n=6 \Rightarrow (a=4)</math>  <math>n=10 \Rightarrow a=4</math>  <math>n=30 \Rightarrow a=8</math></p> <p>Trying <math>n &gt; 2</math> for at least 3 values of <math>n</math>  <math>n=2</math> A1  <math>n=6</math> A1</p> <p>Full argument to show all cases considered B2</p> <p>Checking all answers in * M1</p> <p><math>n=2, a=8</math>  <math>n=6, a=4</math> } only valid answers A2/1/0  -1 for extra case.</p>	<p>(4)</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>B2</p> <p>M1</p> <p>A2/1/0 -1 for extra case.</p>